Vortices in bubble shaped Bose–Einstein condensates

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Surabaya
Guinea
Arafura Sea
Solomon Sea
Timor Sea
Coral Sea
Suva
NORTHERN TERRITORY
QUEENSLAND
Australia
WESTERN AUSTRALIA
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Melbourne
Auckland
Tasman Sea
TASMANIA
New Zealand
THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

Motivation

Experimental realisation of shell geometries

Topologically interesting geometry

Vortex dynamics on curved surfaces

Bowen group: Science **366**, 1480 (2019)

Padavić, Sun, Lannert and Vishveshwara: Phys. Rev. A **102**, 043305 (2020)

Bereta, Caracanhas and Fetter: Phys. Rev. A **103**, 053306 (2021)

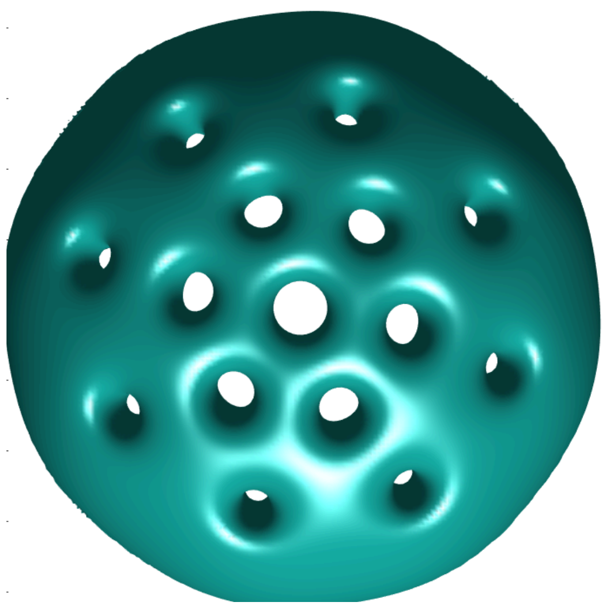
Caracanhas, Massignan and Fetter: Phys. Rev. A **105**, 023307 (2022)

Quantum turbulence on the bubble geometry

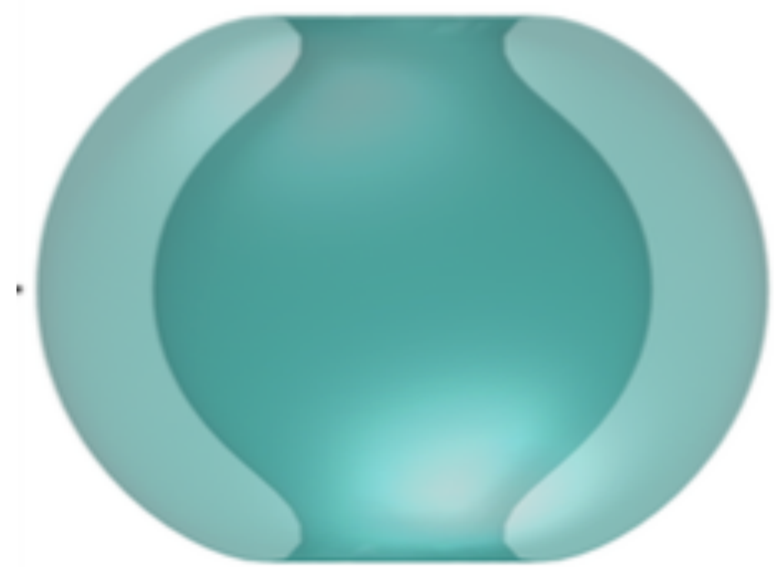
Useful toy models for planetary atmospheric dynamics?

Overview

1. Vortex lattices



2. Multi-charged vortices

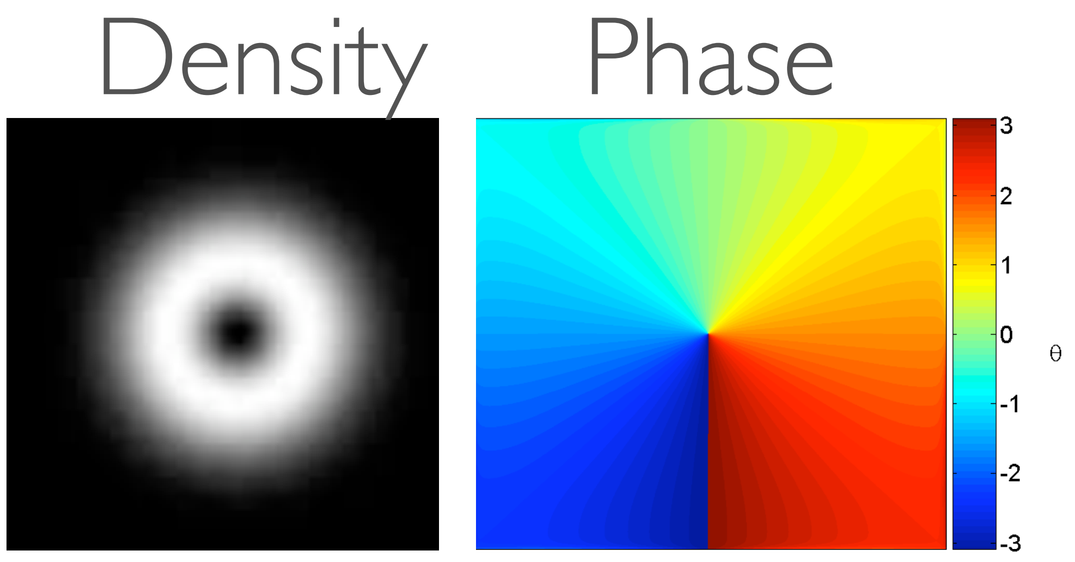


Quantum Rotation

Superfluids:

$$\psi = \sqrt{n}e^{i\theta}$$

$$\mathbf{v} = \frac{\hbar}{m}\nabla\theta$$



$$\oint_C \mathbf{ds}\cdot\mathbf{v} = \frac{\hbar}{m}\oint_C \mathbf{ds}\cdot\nabla\theta = \frac{\hbar}{m}2\pi n$$

$$n = 0, \pm 1, \pm 2...$$

Circulation is either 0 or a multiple of $2\pi\hbar/m$

Topologically Interesting

Hedgehog / hairy ball theorem:

'*We can't have a vector field on a sphere that is both continuous and everywhere non-zero.*'

Impossible to perfectly comb a hairy sphere.

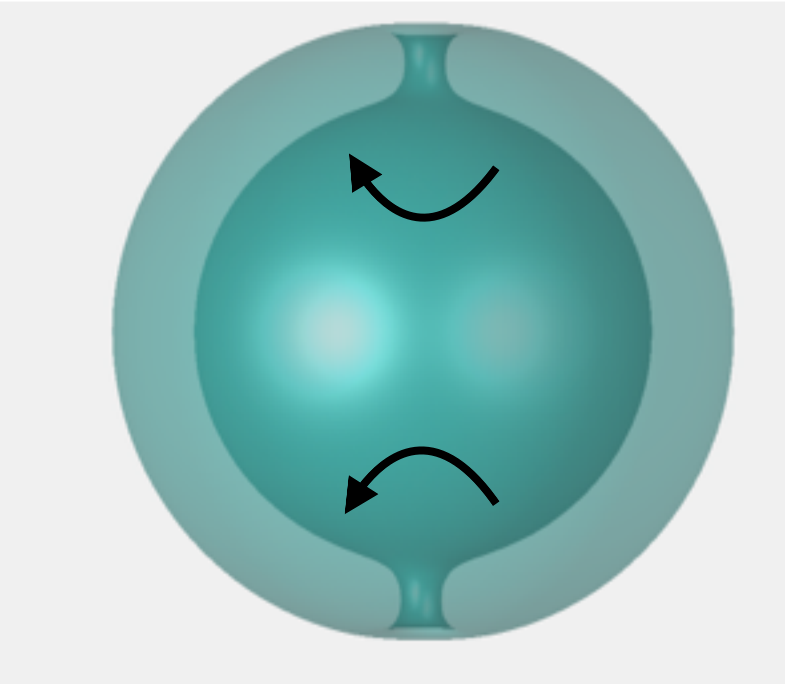
User NoJhan
https://en.wikipedia.org/wiki/Hairy_ball_theorem

Consequence for a BEC on a sphere:

A vortex on a sphere should always come in pairs.

Lamb *Hydrodynamics* Dover publications, New York 1945

A vortex on a sphere should always come in pairs

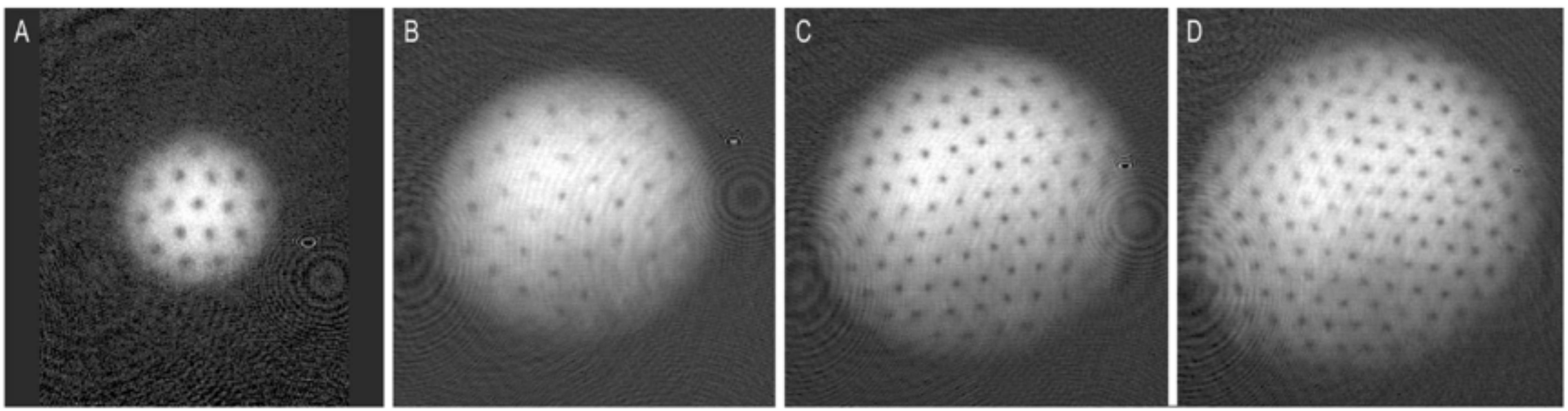


Side view

Top view

How does background curvature change vortex lattice formation?

Triangular ‘Abrikosov’ vortex lattice



16 32 80 130

Ketterle Lab: Science **292** p476-479 (2001)

How does background curvature change vortex lattice formation?

Rotation around z-axis:
do we expect Abrikosov vortex lattice?

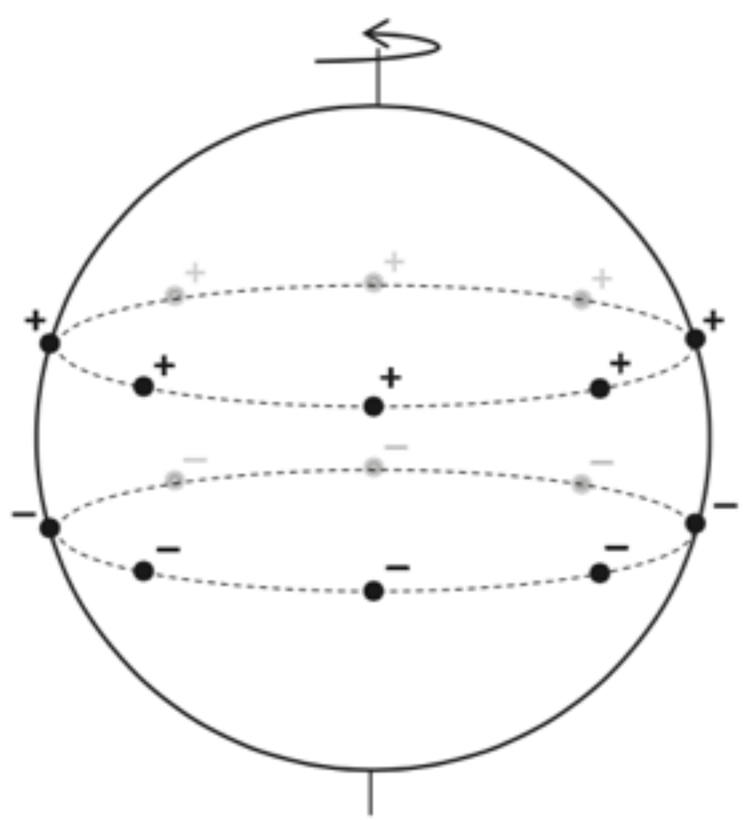


FIG. 1. A rotating $2N$-vortex solution to the point-vortex problem on S^2.

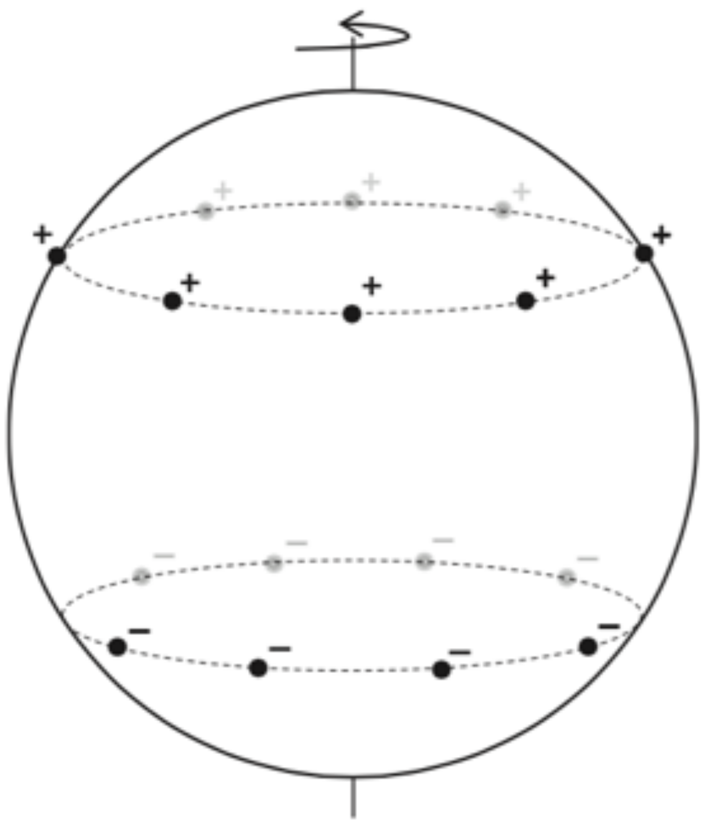


FIG. 2. The skewed rotating $2N$-vortex solution to the point-vortex problem on S^2.

M. Gelantalis & P. Sternberg, J. Math. Phys, **53**, 083701 (2012).

Vortex lattices in a curved background

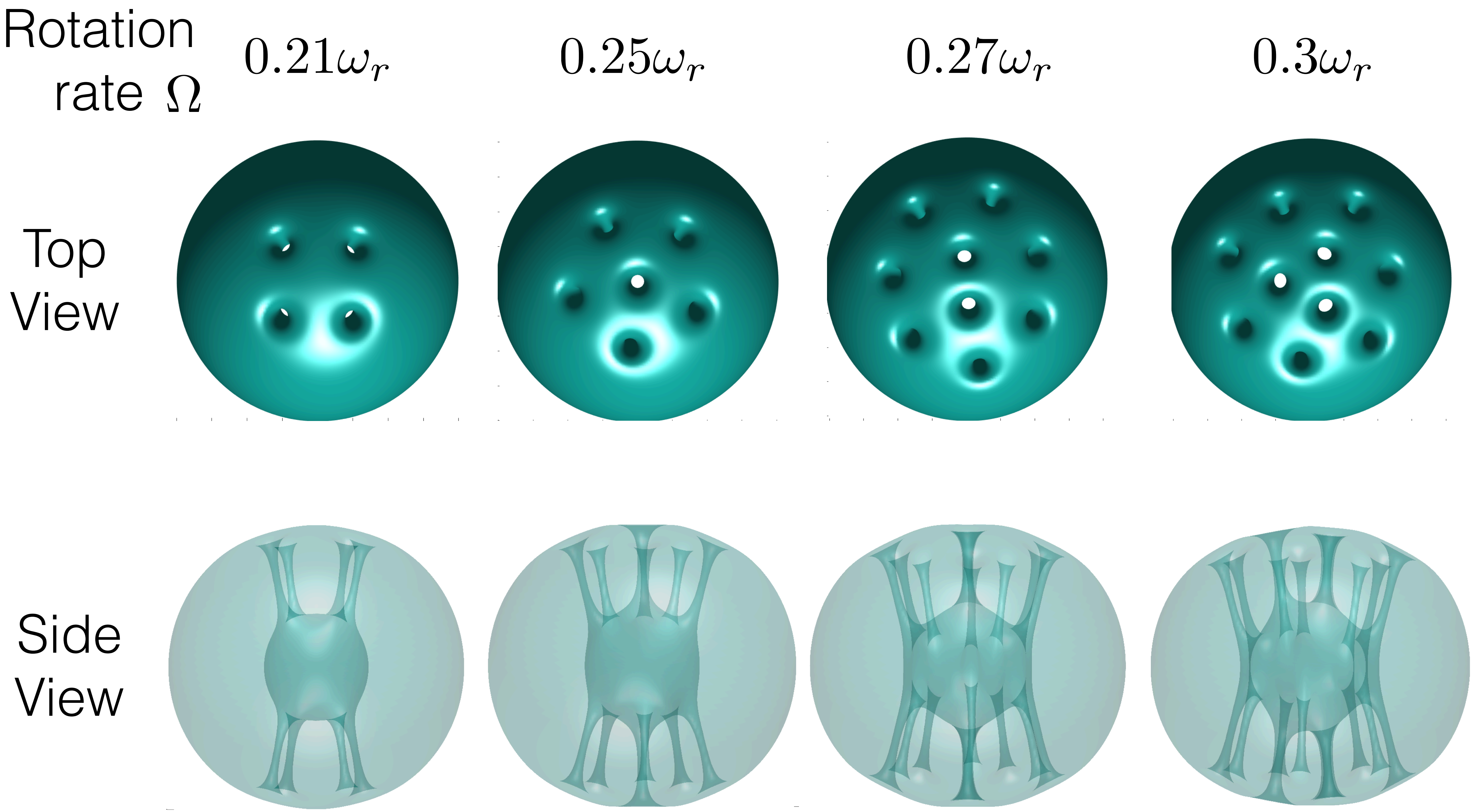
Gross-Pitaevskii equation in the rotating frame:

$$i\hbar\frac{\partial}{\partial t}\psi = \left(-\frac{\hbar^2}{2m}\nabla^2 + V + g|\psi|^2 - \Omega_z L_z\right)\psi$$

Spherical shell potential: $V = \frac{1}{2}m\omega_r^2\left(r - r_0\right)^2$

Angular momentum operator: $L_z = xp_y - yp_x$

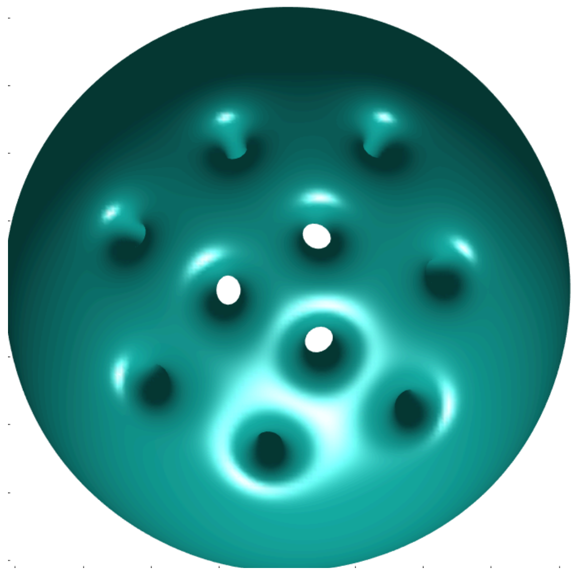
Vortex lattices in a curved background

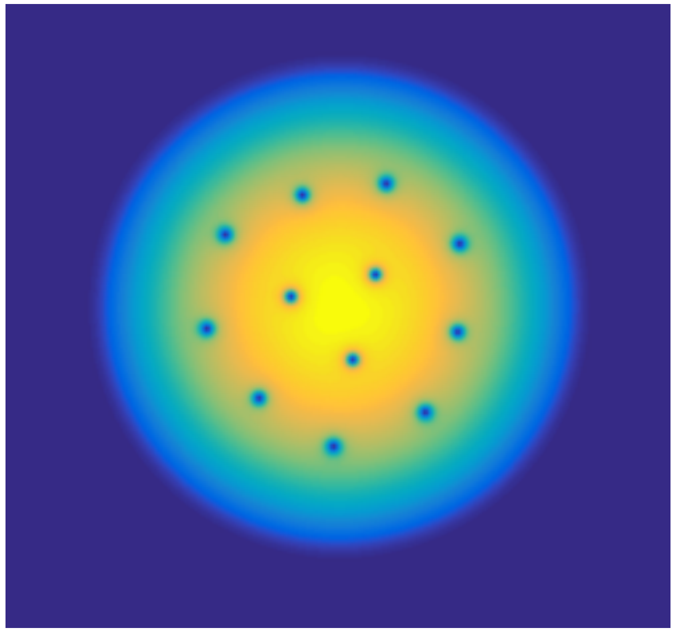


Vortex lattices in a curved background

Low rotation rates $\Omega = 0.3\omega_r$

Top view





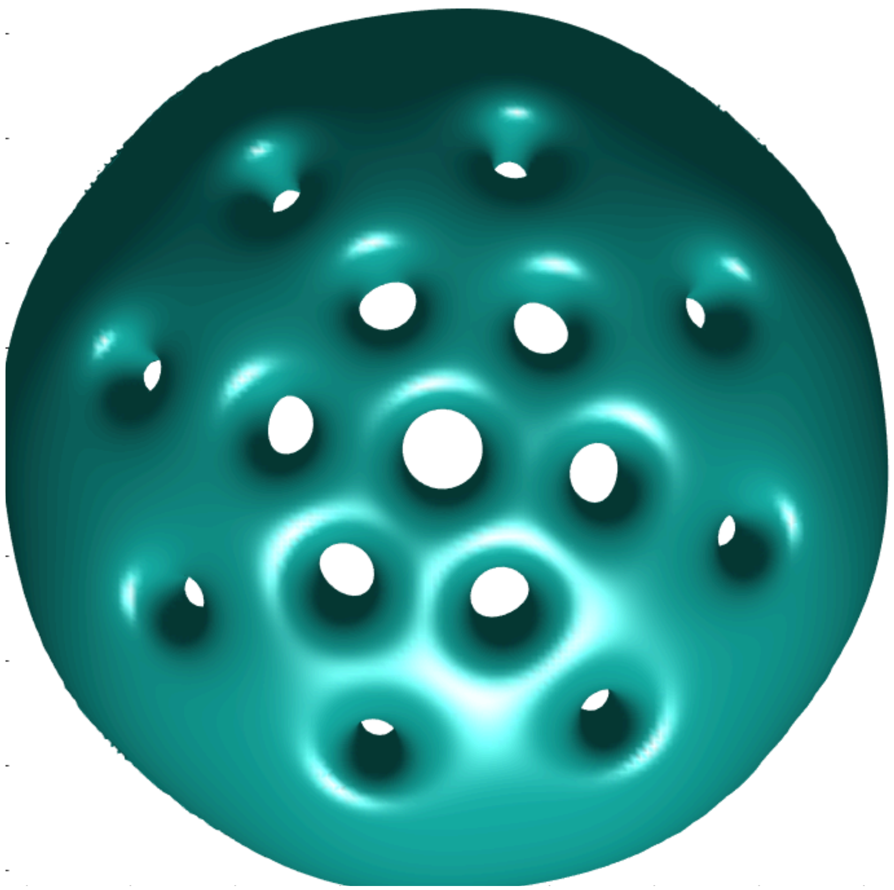
Compare to rotation of a 2D disk-shaped BEC

Abrikosov-like!

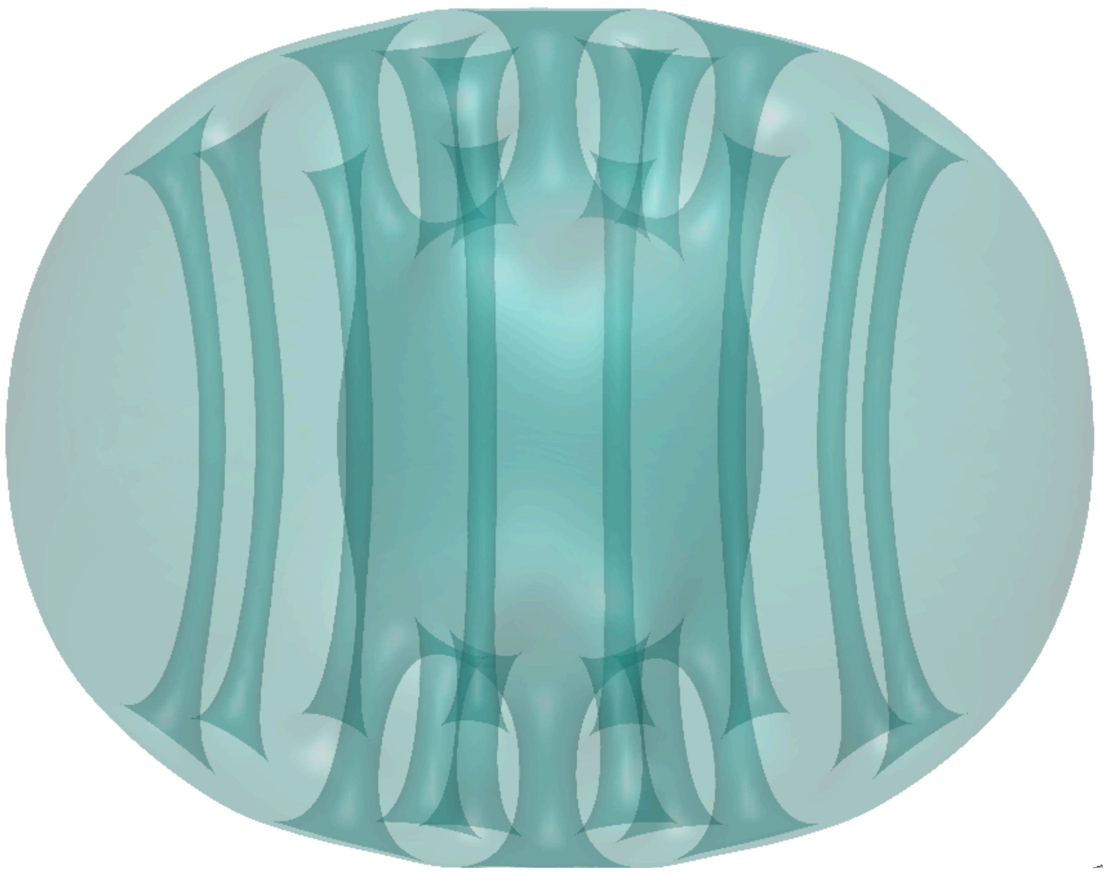
Vortex lattices in a curved background

Faster rotation rates $\Omega = 0.35\omega_r$

Top view



Side view



Transition to vortex lines through the bulk condensate

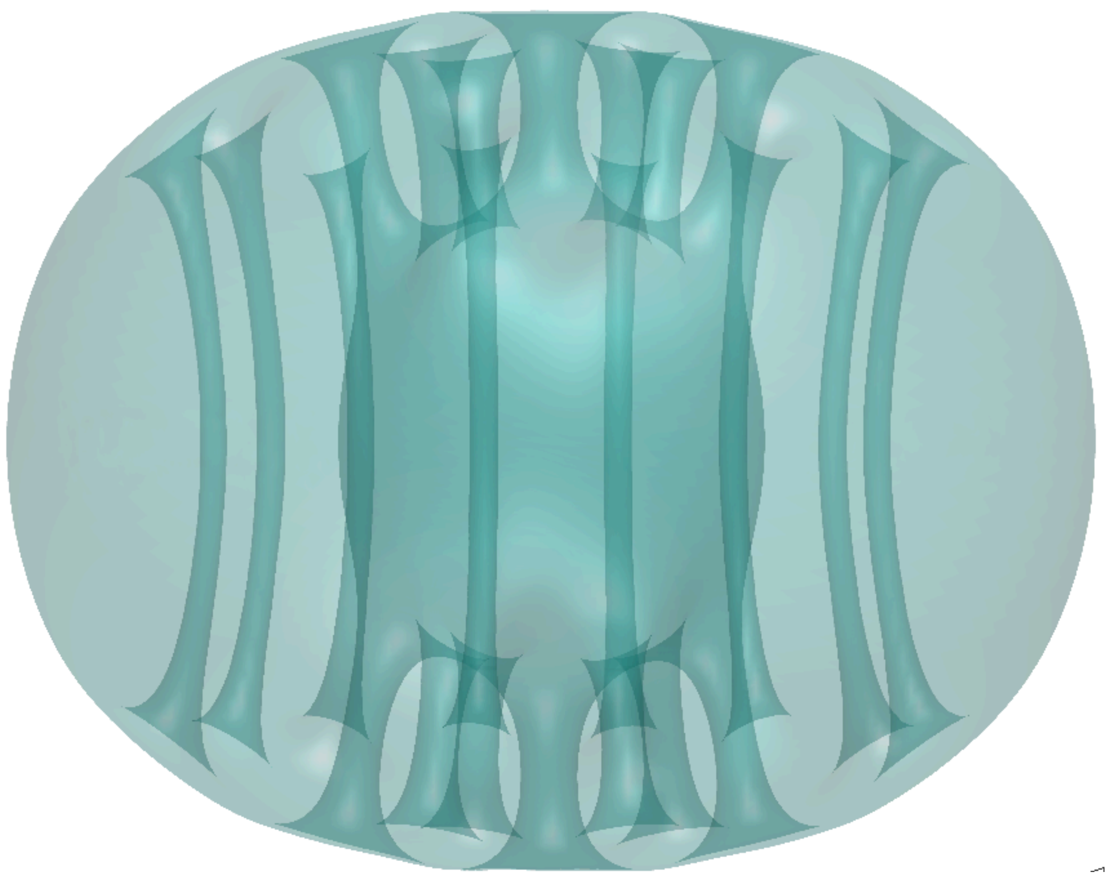
Distortion of the spherical shell shape

Vortex lattices in a curved background

Distortion of the spherical shell shape

$$\Omega = 0.35\omega_r$$

Side view



Centrifugal force:
Pushes atoms away from the centre of rotation

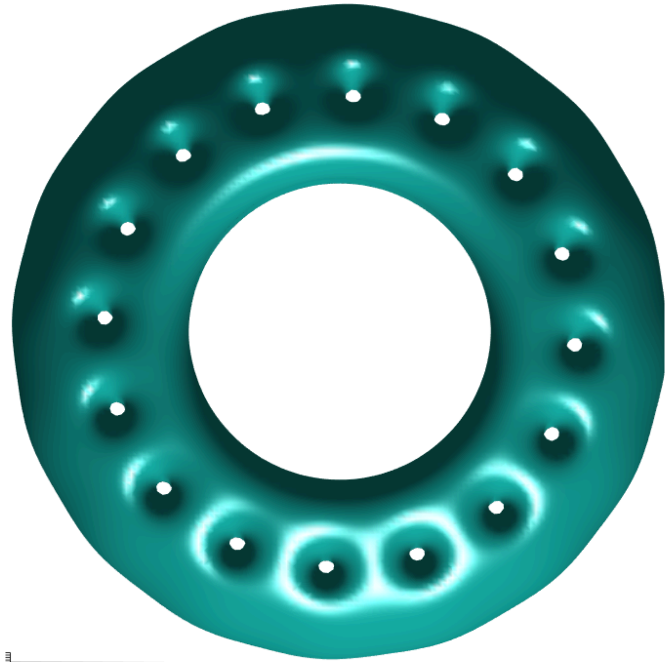
$$n = \frac{1}{U_0}\left(\mu - V + \frac{1}{2}m\left(\Omega \times \mathbf{r}\right)^2\right)$$

Will we observe a transition to toroidal geometries under rotation for thin shells ?

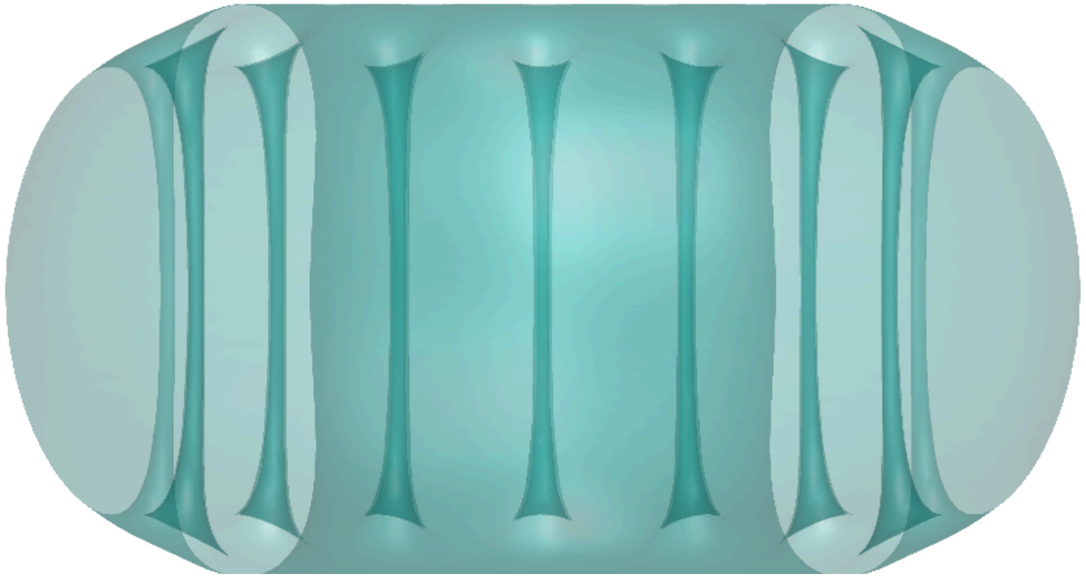
Vortex lattices in a curved background

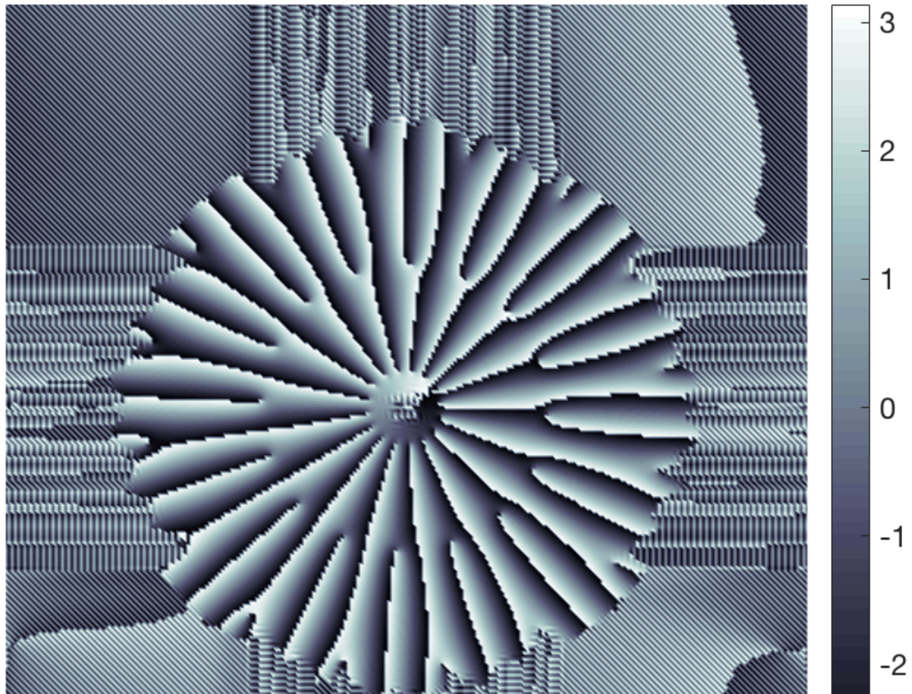
Faster rotation rates $\Omega = 0.5\omega_r$

Top view



Side view





Phase
Top view

Vortex lattices in a curved background

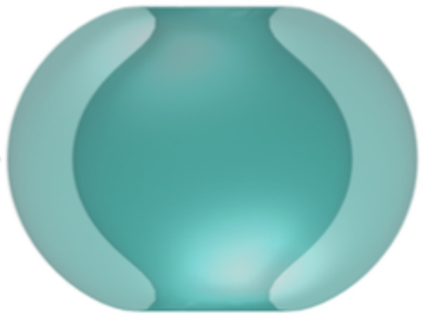
Low rotation rates - familiar triangular lattices emerge

Distortion in the spherical shell shape to elliptical shells due to centrifugal force under rotation

Faster rotation rates - appearance of a large multi-charged vortex core

Future work:
Experimentally realistic potentials

2. Multi-charged vortices



Does the shell geometry provide any topological protection for exotic topological excitations?

Kravchuk et al *Topologically stable magnetisation states on a spherical shell: Curvature stabilised skyrmions* Phys. Rev. B **94** 144402 (2016)

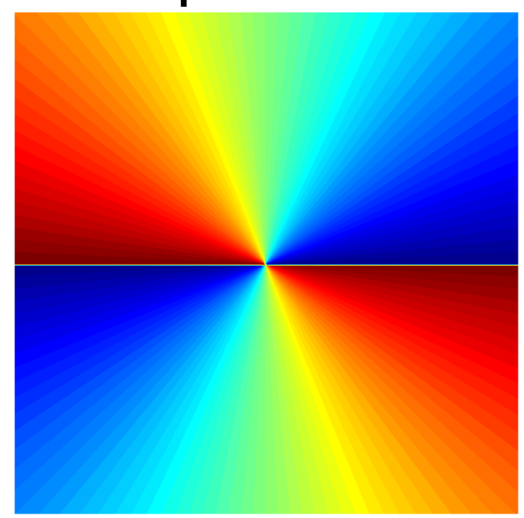
Phase-imprinting multi-charge vortices

Madison Wait

$$\psi = \sqrt{n} e^{il\phi}$$

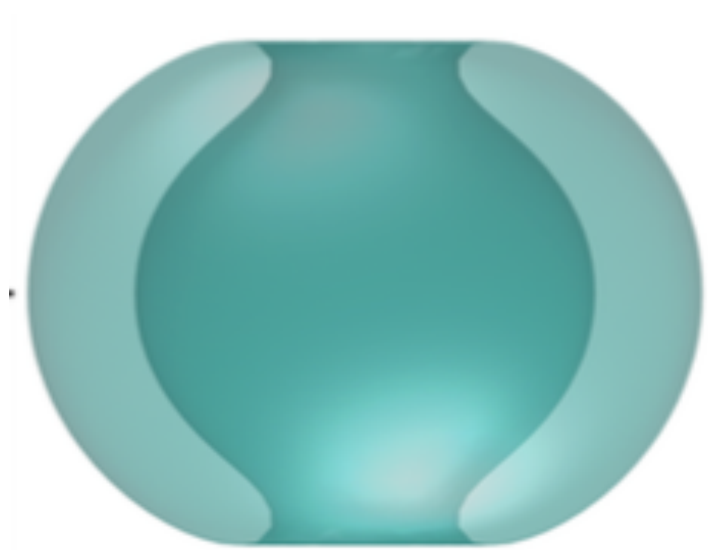
Side view

Top View



phase profile
$l = 2$ vortex

Side view

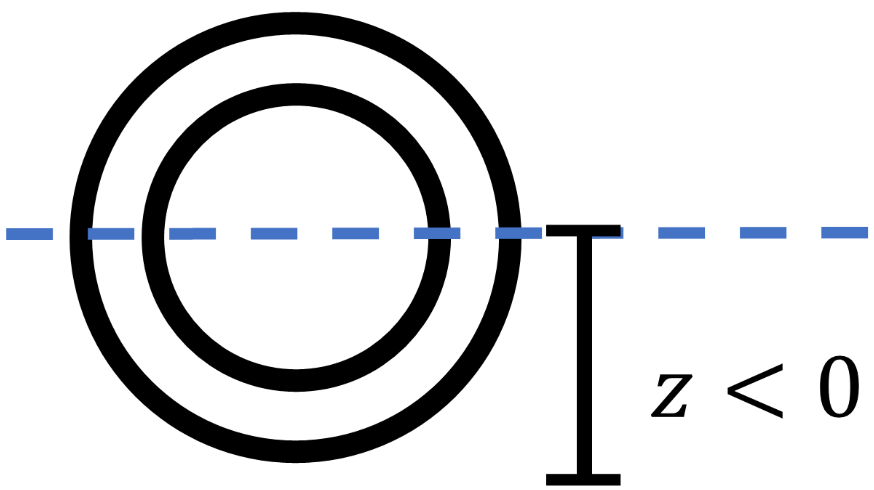


Resulting density profile
$l = 2$ vortex

Multi-charged vortex-anti-vortex pair

Phase-imprinting multi-charge vortices

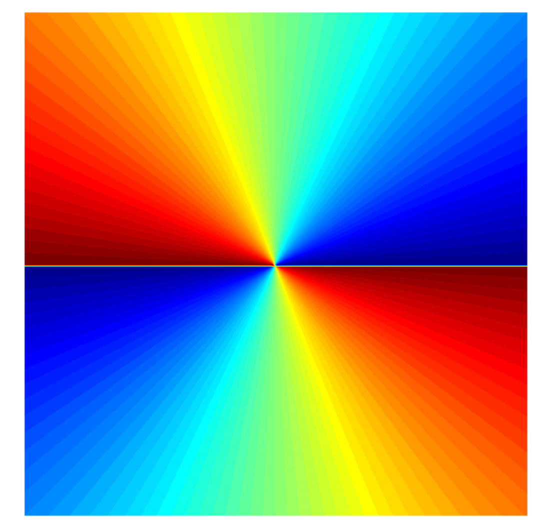
Side view



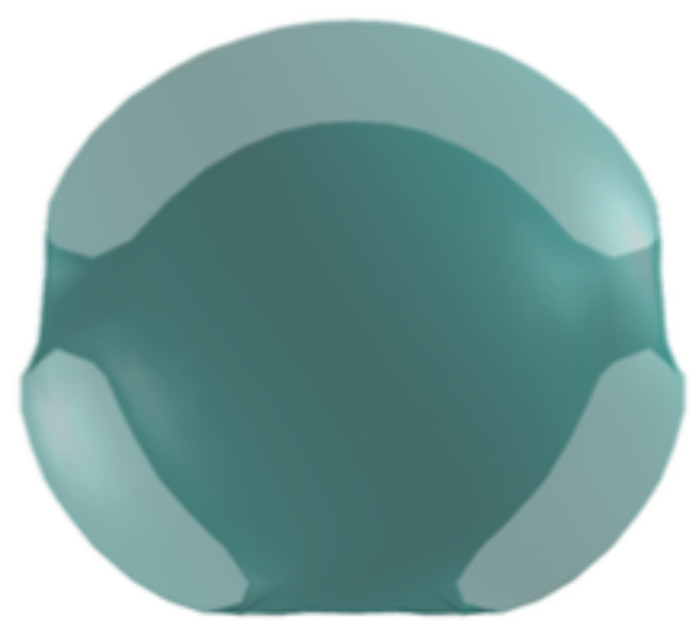
do not enforce phase

enforce phase profile
$l = 2$ vortex

Top View



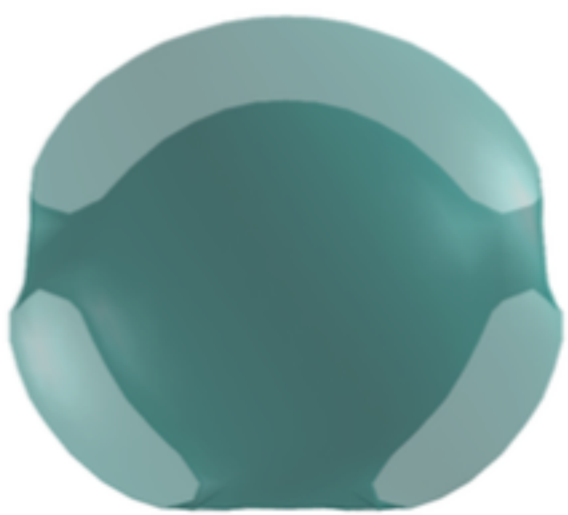
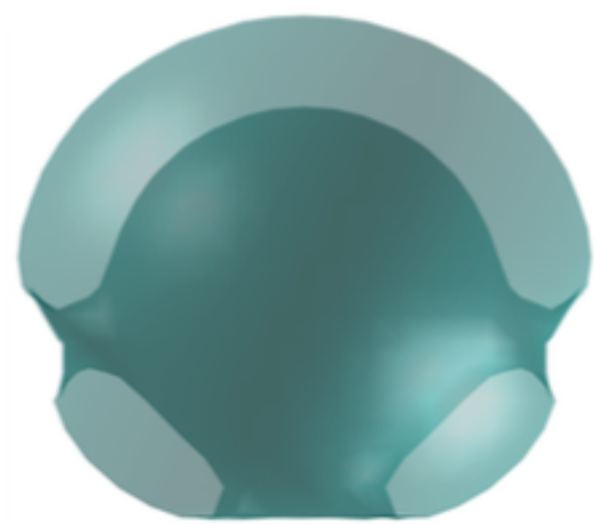
Resulting density isosurface
$l = 2$ vortex and two
singly charged
equatorial vortices



Side view

Phase-imprinting multi-charge vortices

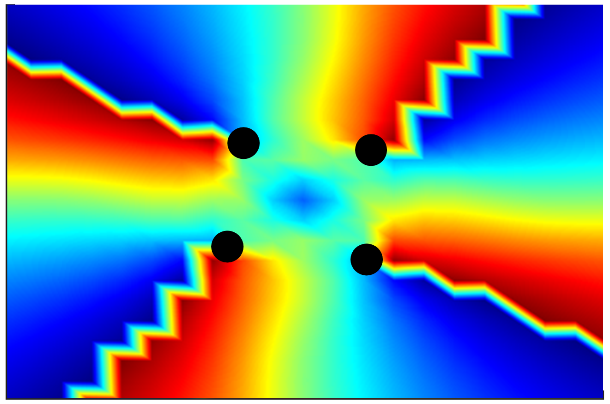
Varying the region or ‘patch’ of phase-imprinting changes the location of singly-charged vortices

Imprinting region	$z < 0$	$z < z_0 < 0$
Side view		

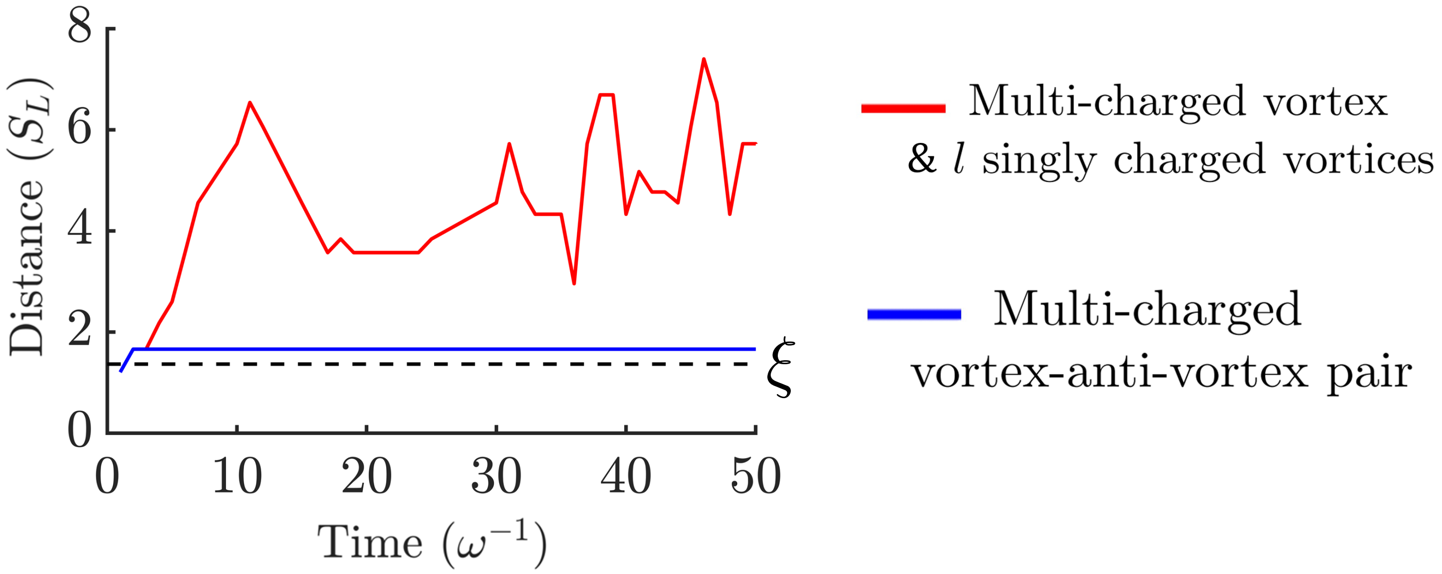
Multi-charge vortex dynamics

Under evolution, a configuration of a multi-charged vortex with singly charged vortices decays.

Phase

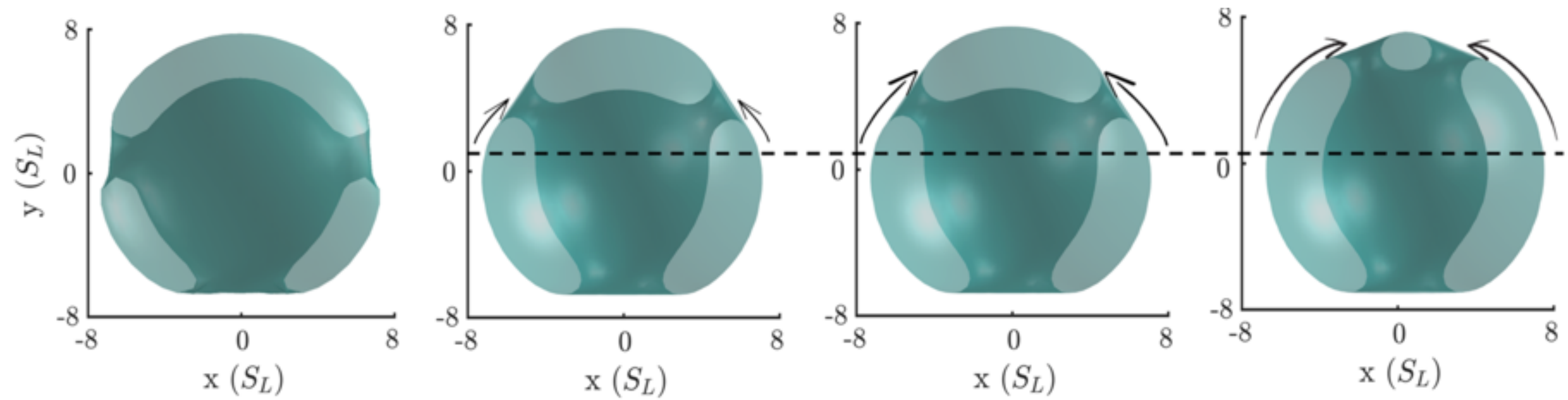


Variation of average separation distance



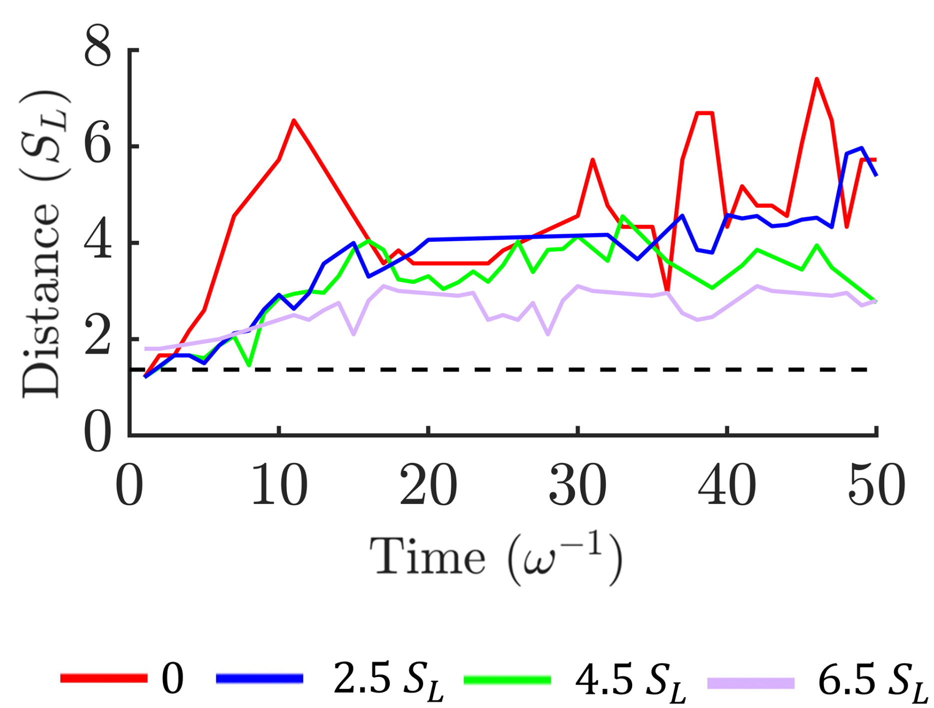
Same behaviour under addition of noise

Moving the singly charged vortices towards the antipode



2.5 S_L 4.5 S_L 6.5 S_L

Average vortex separation distance $(l = 4)$

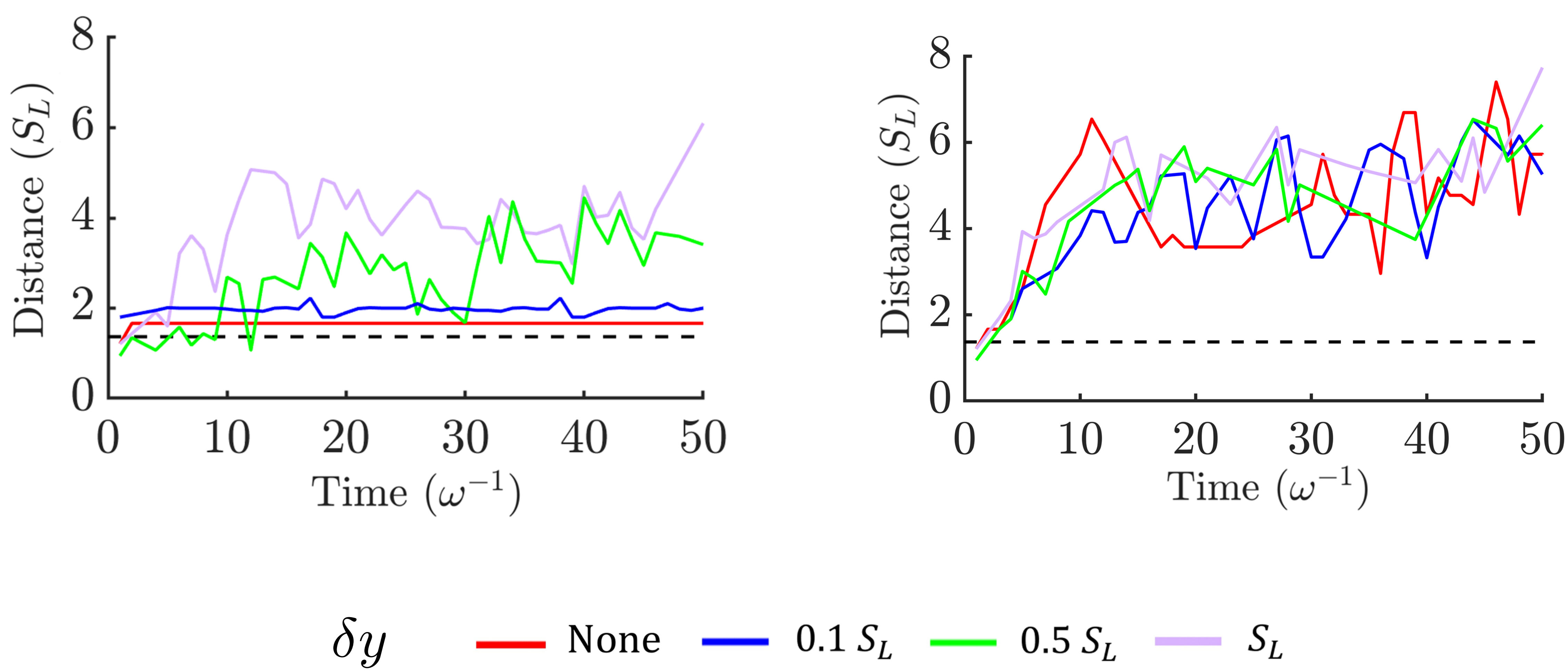


Forcing decay by a sudden asymmetric shift in trap potential

$$V = \frac{(r - r_0)^2}{2} \quad \rightarrow \quad \frac{\left(\sqrt{x^2 + (y + \delta y)^2 + z^2} - r_0\right)^2}{2}$$

$l = 4$ Multi-charge vortex - antivortex pair

$l = 4$ Multi-charge vortex with four singly charged equatorial vortices



Stability of multi-charged vortices

Madison Wait

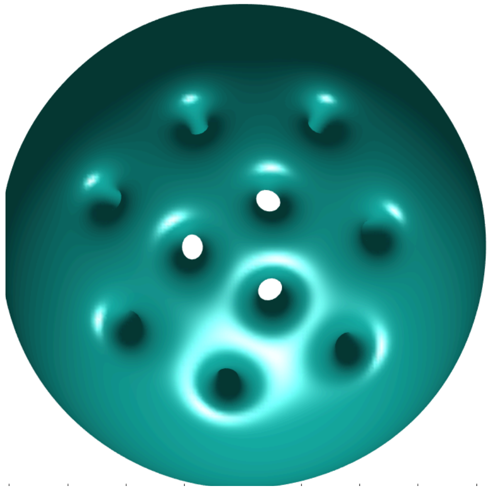
Multi-charged vortex with singly charged vortices
is unstable to decay

Multi-charged vortex - anti vortex pair
at the antipode appears stable to unwinding
except for a large sudden asymmetric shift in
the trapping potential

Is this enhanced stability due to topology?

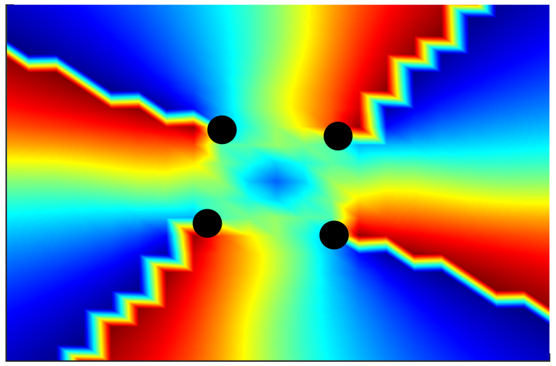
Is there 'topological protection' of other exotic topological states?

Conclusions



Familiar triangular Abrikosov lattices appear for shell condensates under rotation

Centrifugal force distorts spherically symmetric shells to elliptical shapes



Multi-charged vortex pairs at the antipodes appear to be stable.

A multi-charged vortex will readily decay paired with singly charged vortices at the antipode.

Thank you!