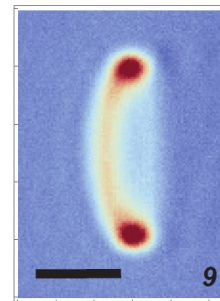


Condensate Bubbles: Co-existence, Collective Modes, Cooling and more



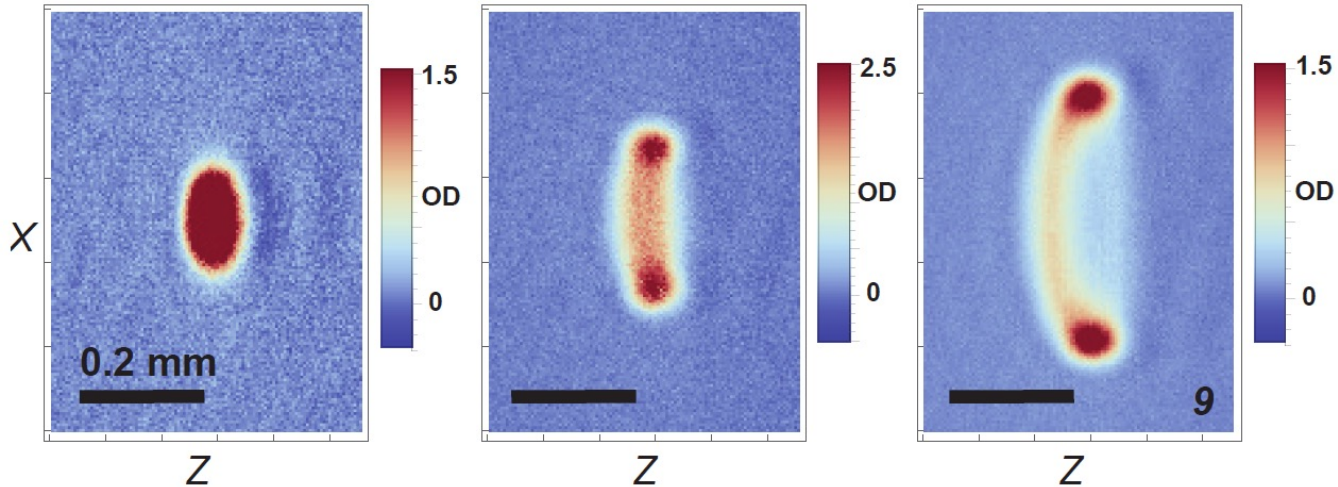
Smitha Vishveshwara
Dept. of Physics, University of Illinois at Urbana-Champaign



In Collaboration with
Groups of *C. Lannert, N. Lundblad, JPL*
K. Sun, K. Padavic, B. Rhyno,
D. Aveline, R. Barankov, B. DeMarco, R. Carollo,
J. Murphee, T.C. Wei, F. Yang

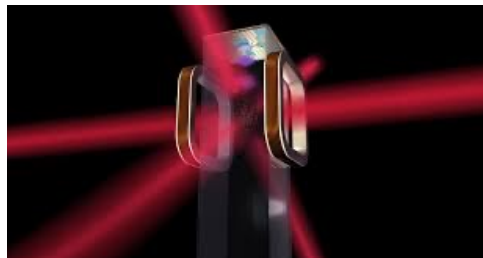
Ap7, 2022; Workshop, Prospects of Quantum Bubble Physics, Leibniz Univ.

Observation and Thermodynamics of Quantum Bubbles in Space



Lundblad Group+JPL
CAL, ISS
arXiv2108:05880
Accepted to Nature

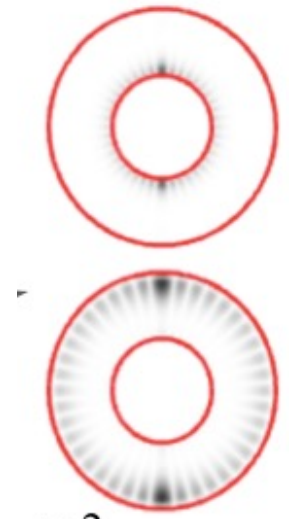
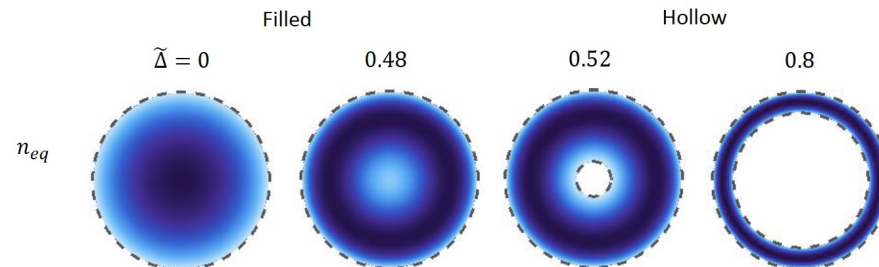
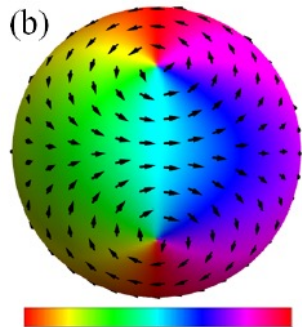
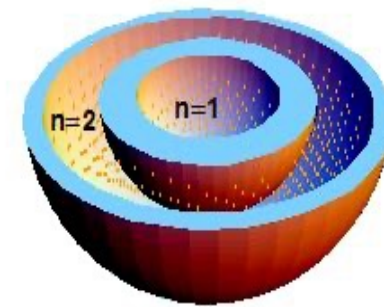
*Theoretical Input (Rhyno)
along with Lannert Group*



Talks by Lundblad; Garroway, Salasnich, von Klitzing, and Williams

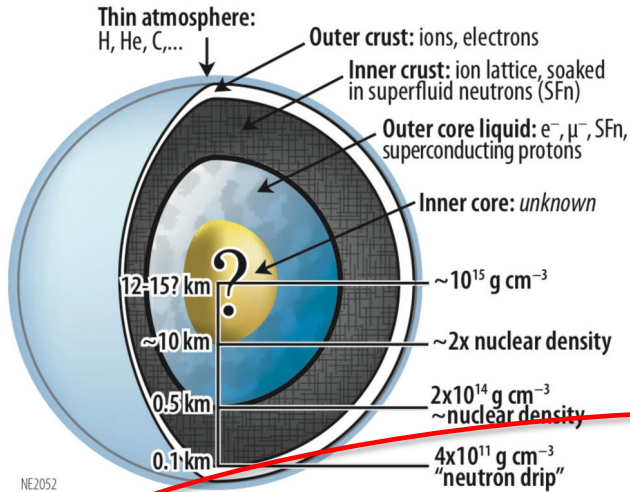
In what follows

- Condensate shells in different contexts
- Superfluid-Mott systems—realization, collective modes
- Shell expansion
- Connecting with CAL Experiments
- Collective modes contd.
- Vortices
- Thermometry

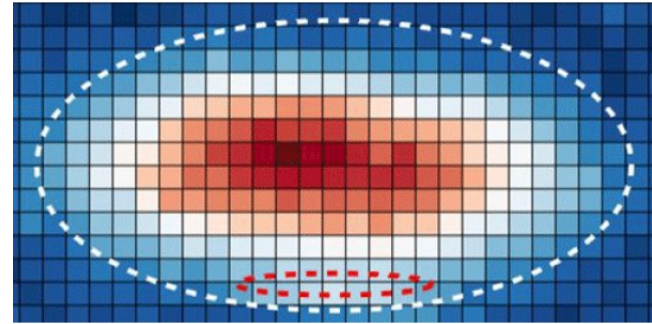


Condensate bubbles in different settings

Stellar Bodies



Gas Mixtures

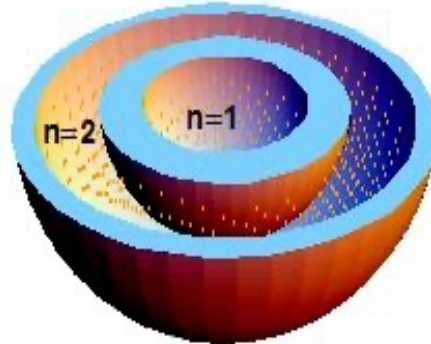


Bose-Fermi: Chin Lab; Bosons—Talk by Meister

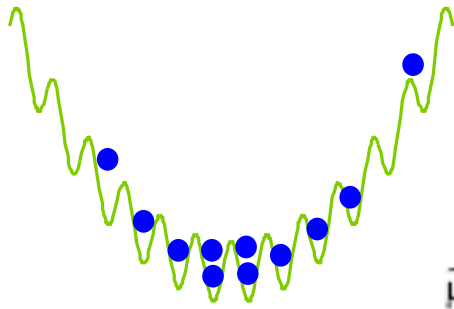
Free Space



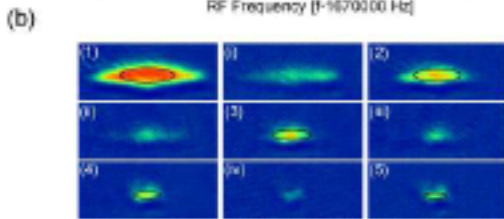
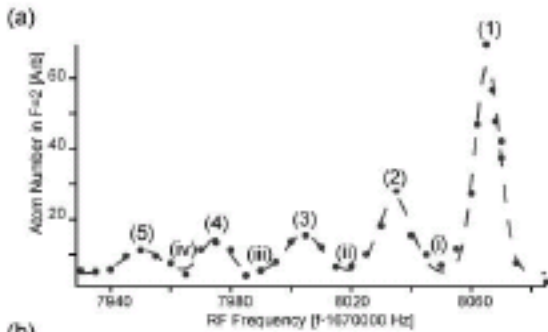
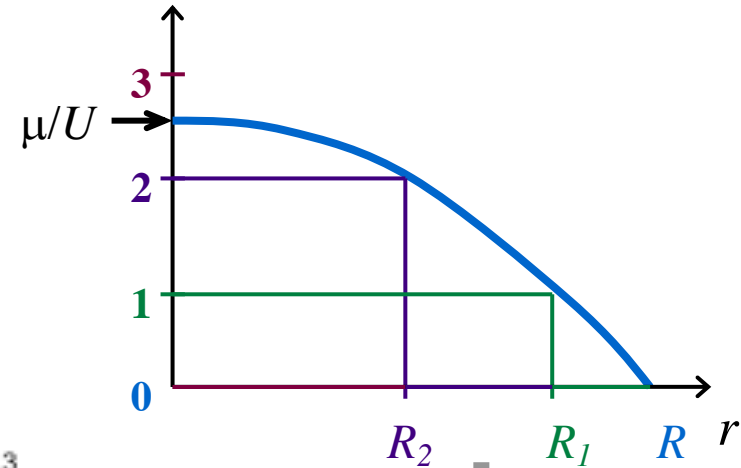
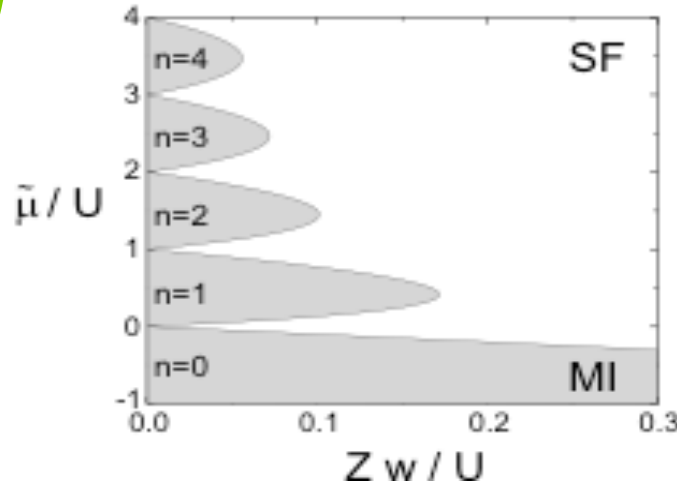
Optical Lattice



Optical Lattices---Coexistence---Mott Insulator Shells

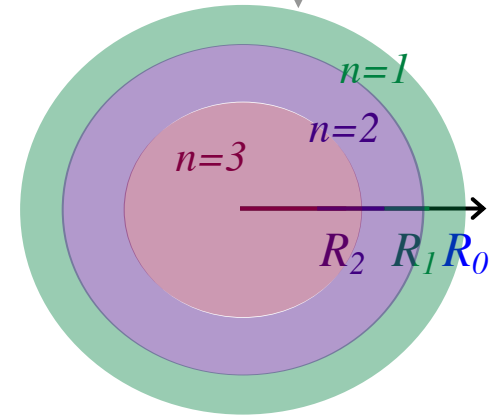


**Optical Lattice+
Harmonic Trap**



Spectroscopy

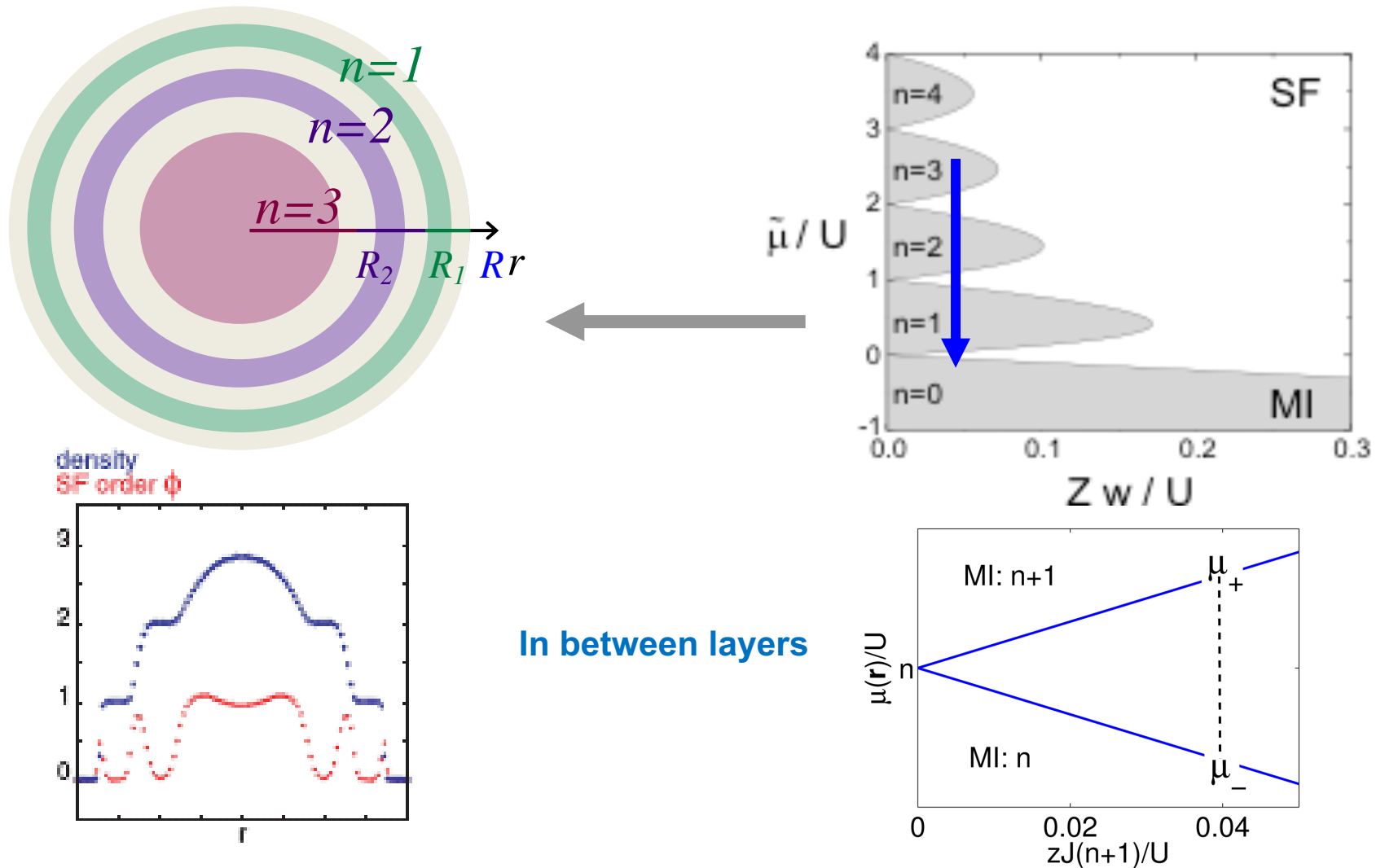
**Low tunneling:
Mott Shells**



G. G. Bartoumi et al PRL (1990) D. Jaksch et al., PRL (1998) ;
De Marco, C. Lannert, SV, T.C. Wei, PRA (2005);

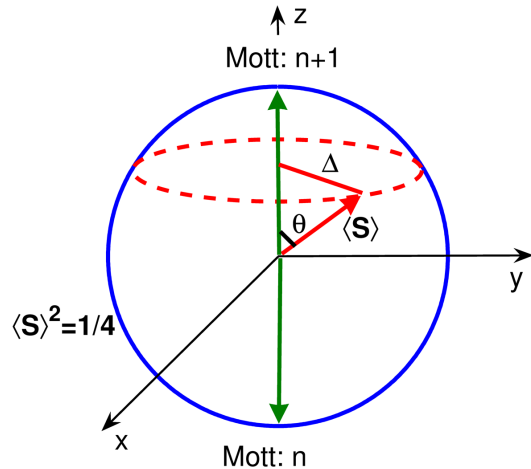
E.g. Campbell et al., Science 2006; Foelling et al., PRL 2006

Number fluctuations – The “wedding cake” scenario



Condensate layer---Collective Modes

Pseudospin description



Density distortions

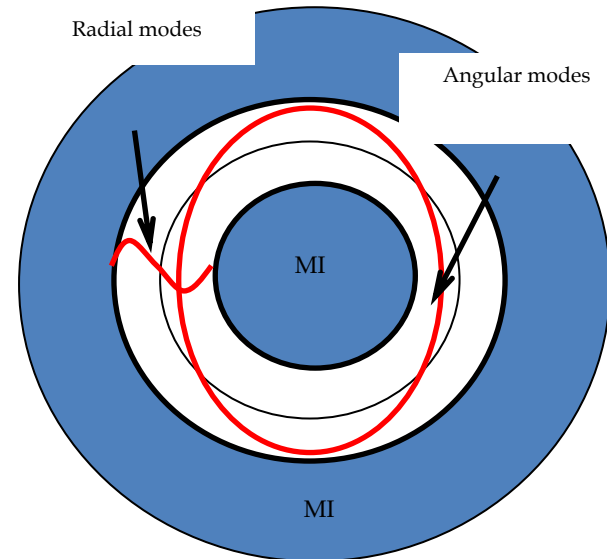
$$\partial_t^2 \delta\rho = 4z (J(n+1)\ell)^2 \nabla \left[\Delta_0^2 \nabla \delta\rho \right]$$

$$\Delta_0(\mathbf{r}) = (1/2) \sin \theta(\mathbf{r})$$

Collective Modes

Radial modes: $\Omega_r = 2\sqrt{6}J(n+1)\ell/\delta R_n$

Angular modes: $\Omega_s = \sqrt{3}J(n+1)\ell/R_n$

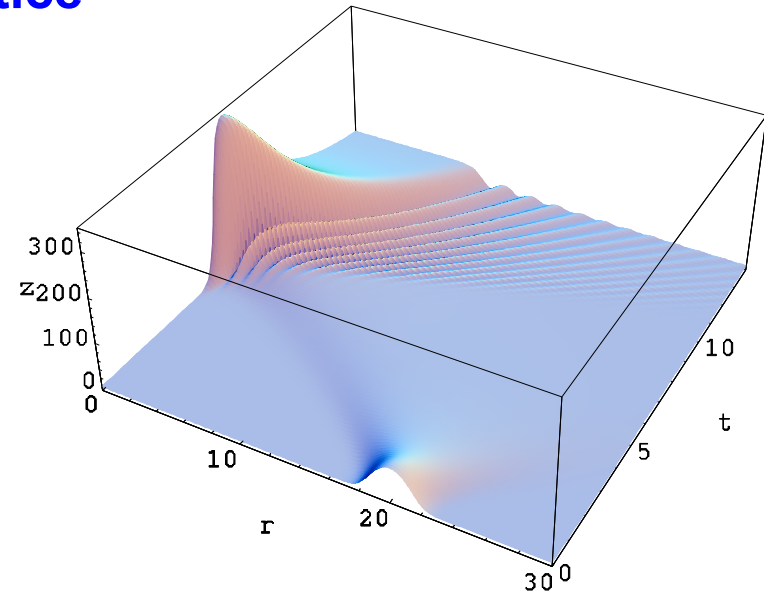
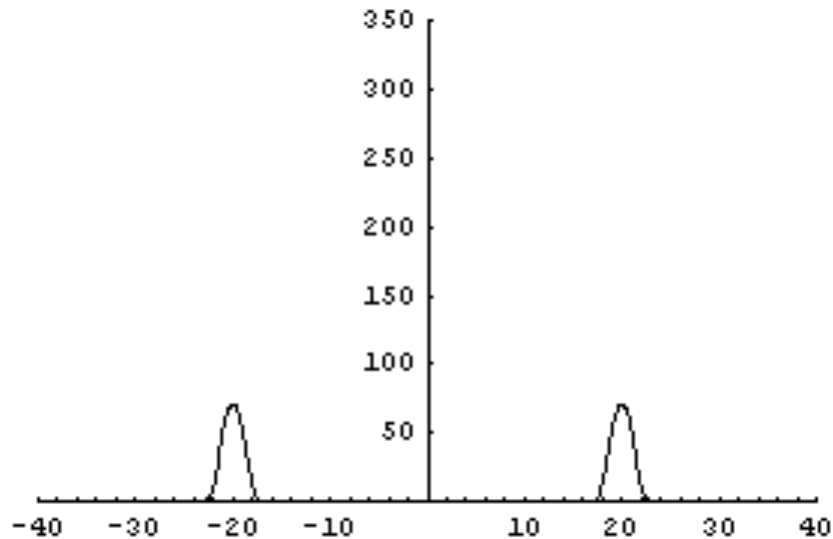


Can probe in spectroscopy

Effective dimensionality of condensate : tunable, determined by Ω_r and Ω_s

Free space—3D shell expansion

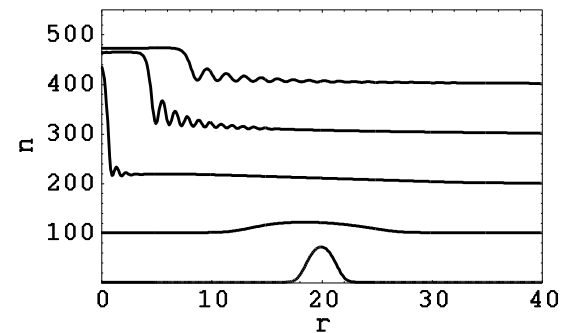
Simulations in the absence of a lattice



Features:
Mass accumulation, self-interference

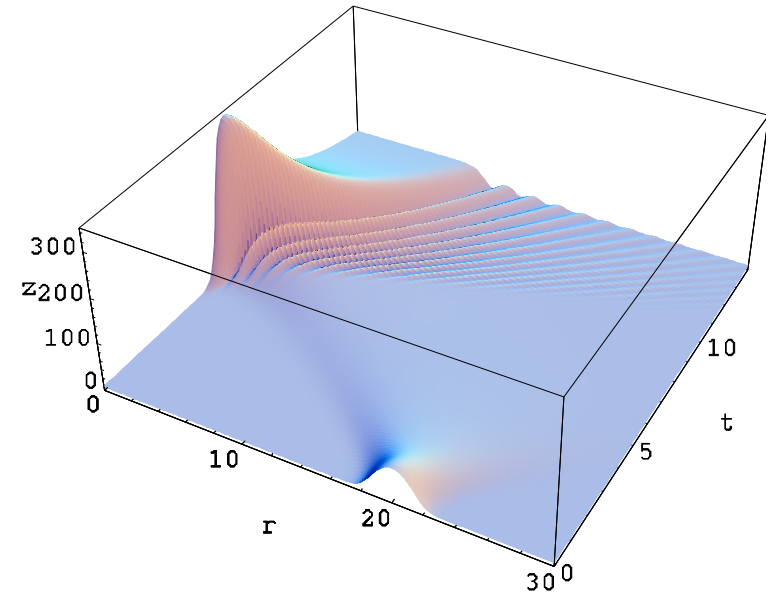
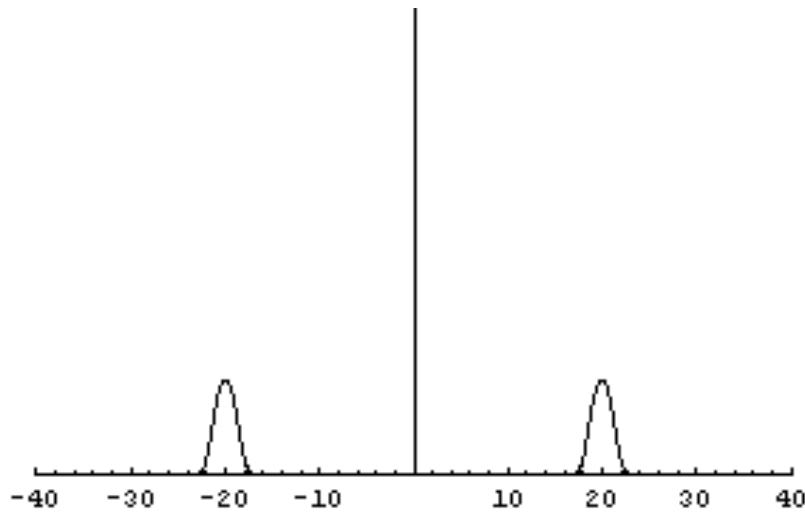
C. Lannert, S.V. and T. C. Wei, PRA (2007)

Related: Tononi, F. Cinti and L. Salasnich, PRL (2020)

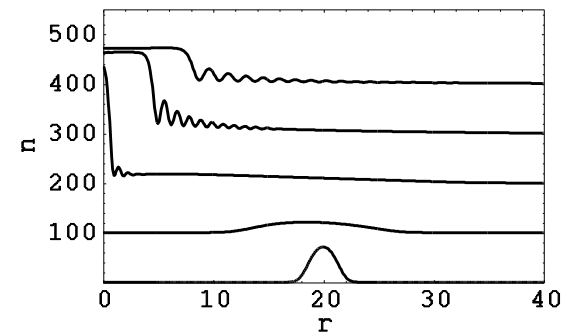


Free space—3D shell expansion

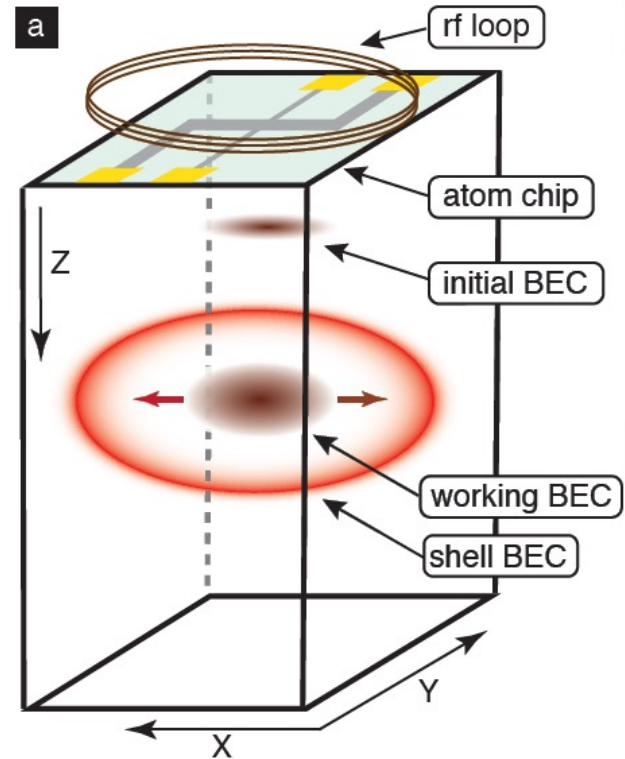
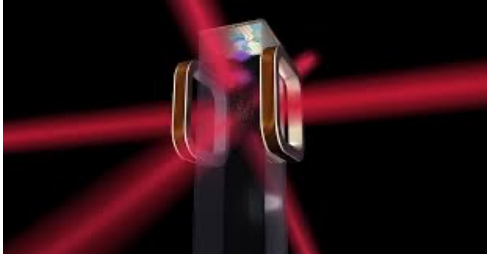
Simulations in the absence of a lattice



Features:
Mass accumulation, self-interference



Bubbles in Space



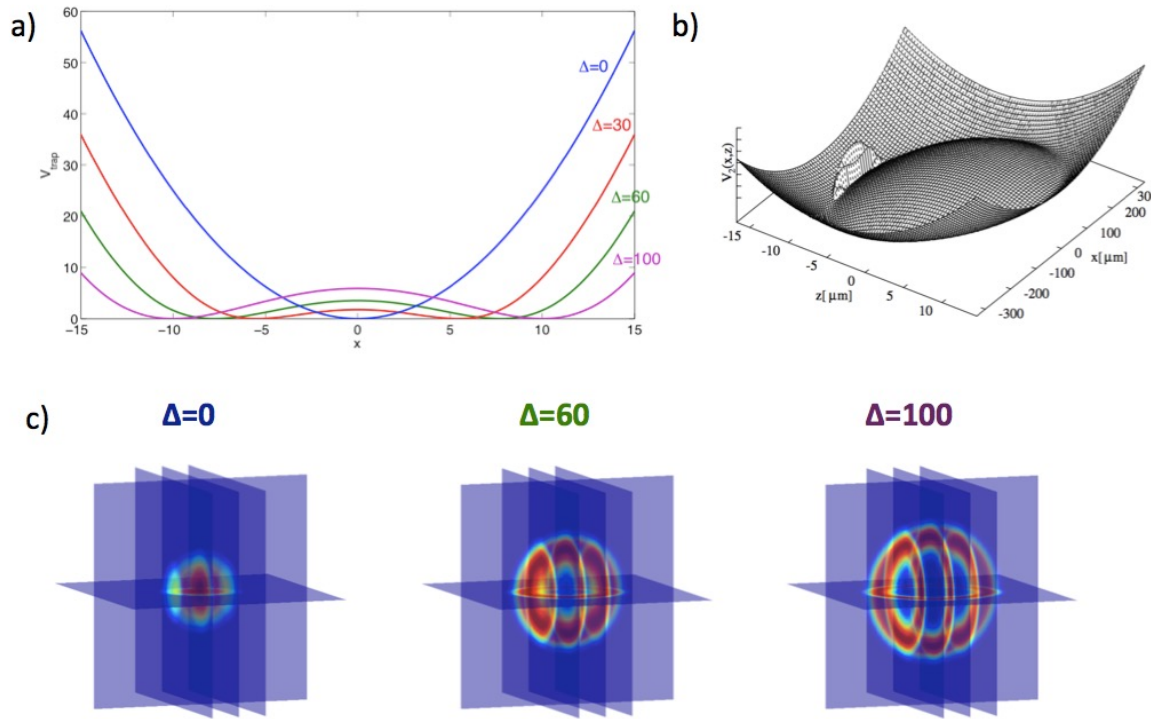
Lundblad et al, NPJ Microgravity (2019)

SPACE & PHYSICS

Ultracold Quantum Collisions Have Been Achieved in Space for the First Time

K. Padavic, Scientific American, March 2021

Condensates in Bubble Traps

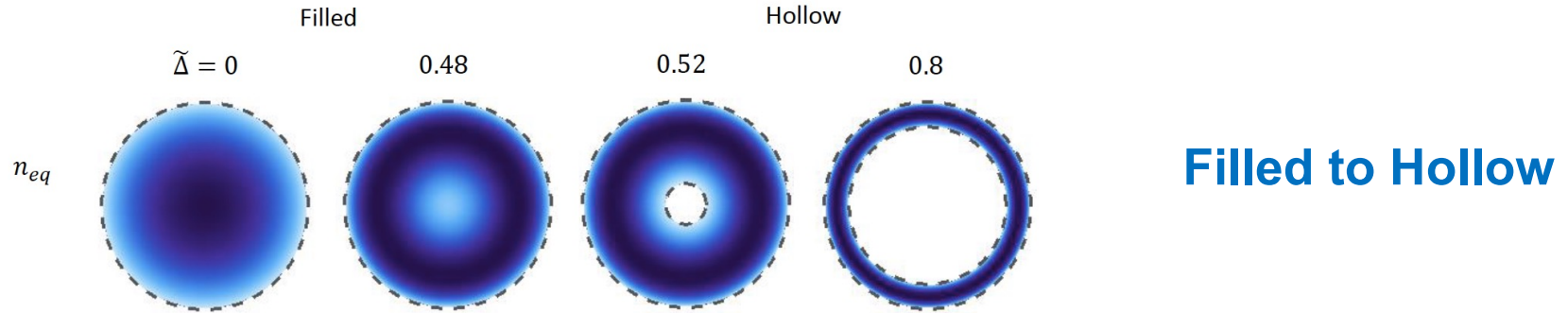


$$V_{\text{bubble}}(\mathbf{r}) = m\omega_0^2 S_l^2 \sqrt{(r^2 - \Delta)^2/4 + \Omega^2}$$

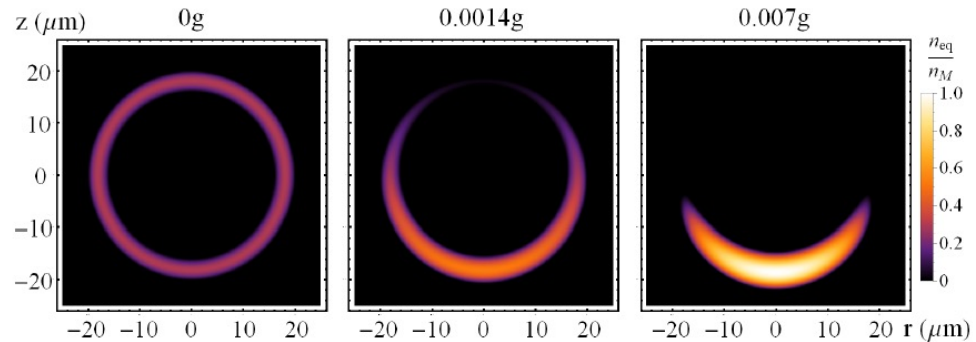
O. Zobay, B.M. Garraway, Phys. Rev. A, (2004); K. Merloti et al, New J. Phys. (2013);

Talk by Garraway

Condensates in Bubble Traps



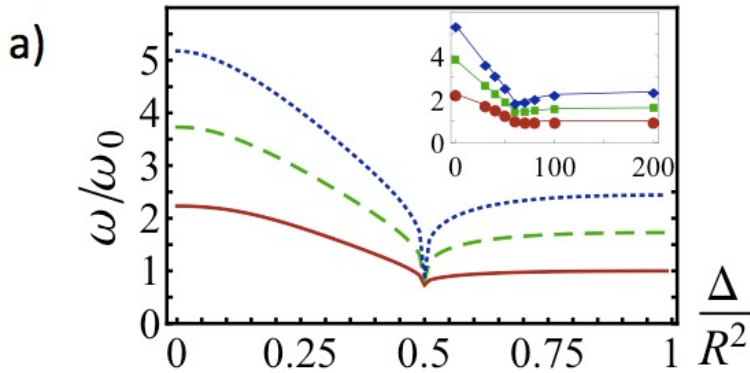
Gravity



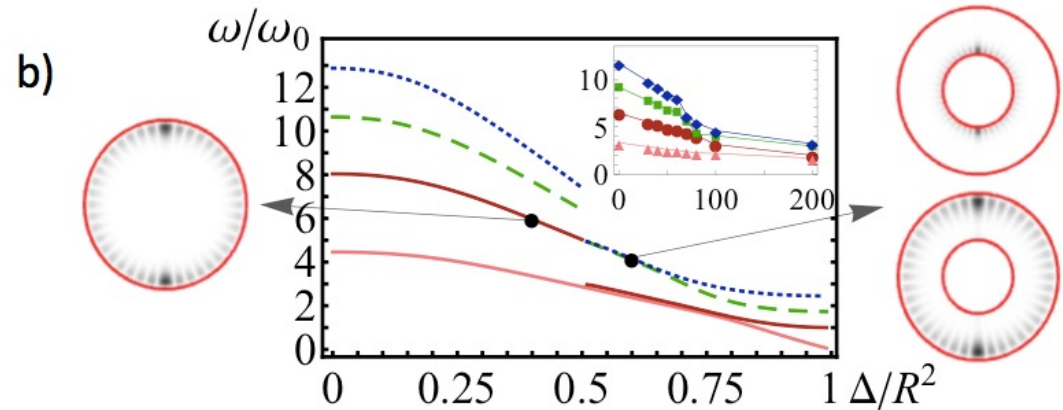
$$V_{\text{bubble}}(\mathbf{r}) = m\omega_0^2 S_l^2 \sqrt{(r^2 - \Delta)^2/4 + \Omega^2}$$

Collective modes and hollowing

Radial Modes ($l=0$)



Surface Modes (large l)



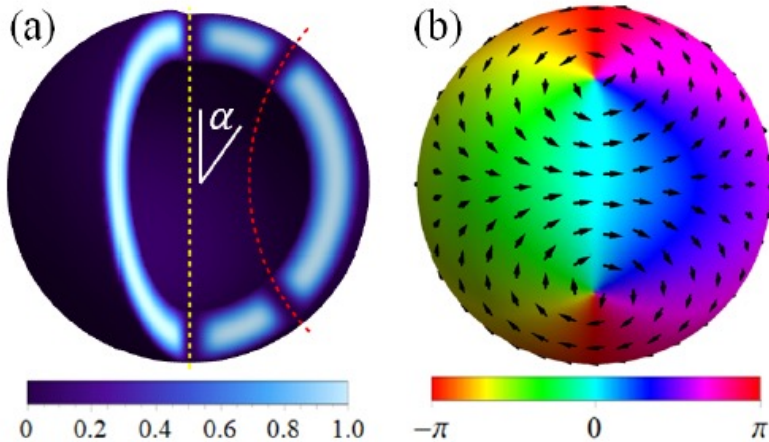
Hollowing-out topological transition:

- Dip in spherically symmetric modes
- Jump in surface mode frequencies

$$-m\omega^2 \delta n = U_0 (\nabla n_{\text{eq}} \cdot \nabla \delta n + n_{\text{eq}} \nabla^2 \delta n)$$

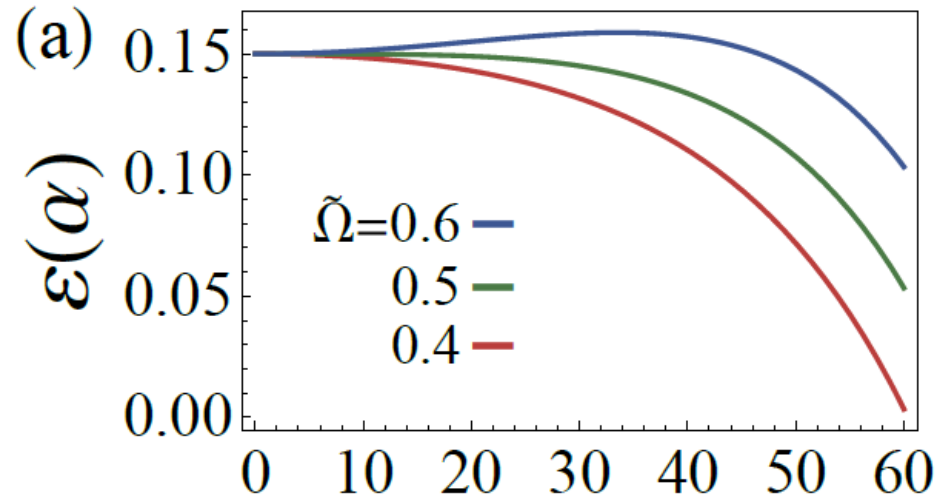
$$V_{\text{bubble}}(\mathbf{r}) = m\omega_0^2 S_l^2 \sqrt{(r^2 - \Delta)^2/4 + \Omega^2}$$

Vortex-Antivortex structure



Simplest
Topologically allowed structure

Energy (dimensionless) as a function
Of vortex line angle
Shows critical angle for stabilization
(2D Limit shown here; 3D systematically studied)



Fetter, RMP (2009)

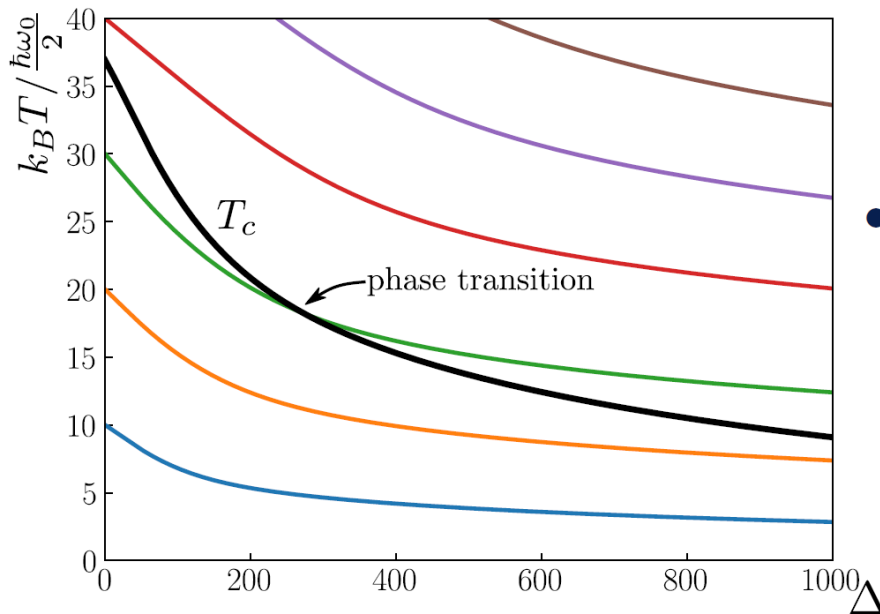
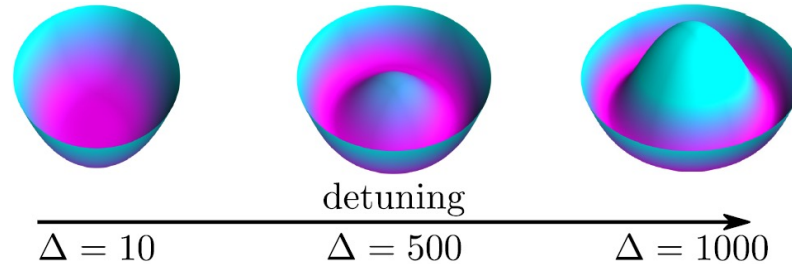
Zhang et al, Phys Rev. Fluids (2018)

Relevant talks by White, Massignan and Dubessy

K. Padavic et al, PRA (2020)

Thermometry

$$V_{\text{bubble}}(\vec{x}) \propto \sqrt{[|\vec{x}|^2 - \Delta]^2 + (2\Omega)^2}$$



- BEC critical temperature:

$$N = \sum_{\alpha \neq 0} \frac{1}{e^{(\varepsilon_{\alpha} - \varepsilon_0)/k_B T_c} - 1}$$

- Temperature as system inflates into a bubble adiabatically:

$$N(T, \mu) = \sum_{\alpha} f_{\alpha}$$

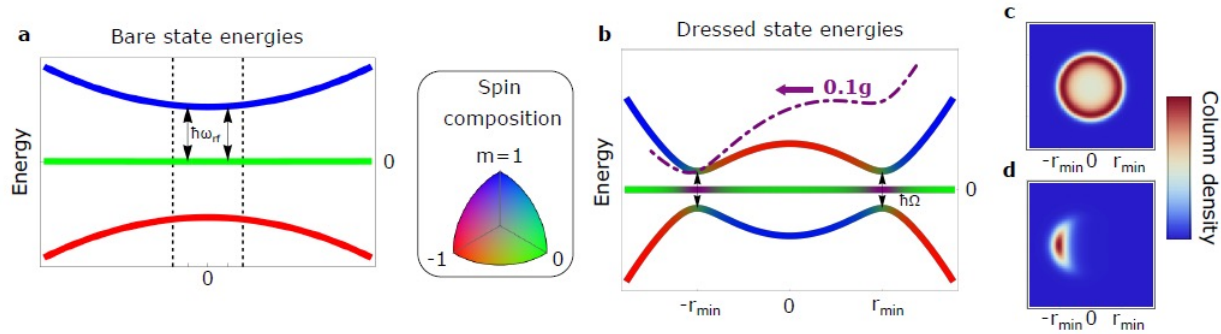
$$S(T, \mu) = k_B \sum_{\alpha} [(1 + f_{\alpha}) \ln(1 + f_{\alpha}) - f_{\alpha} \ln f_{\alpha}]$$

where $f_{\alpha} = 1/(e^{\beta(\varepsilon_{\alpha} - \mu)} - 1)$.

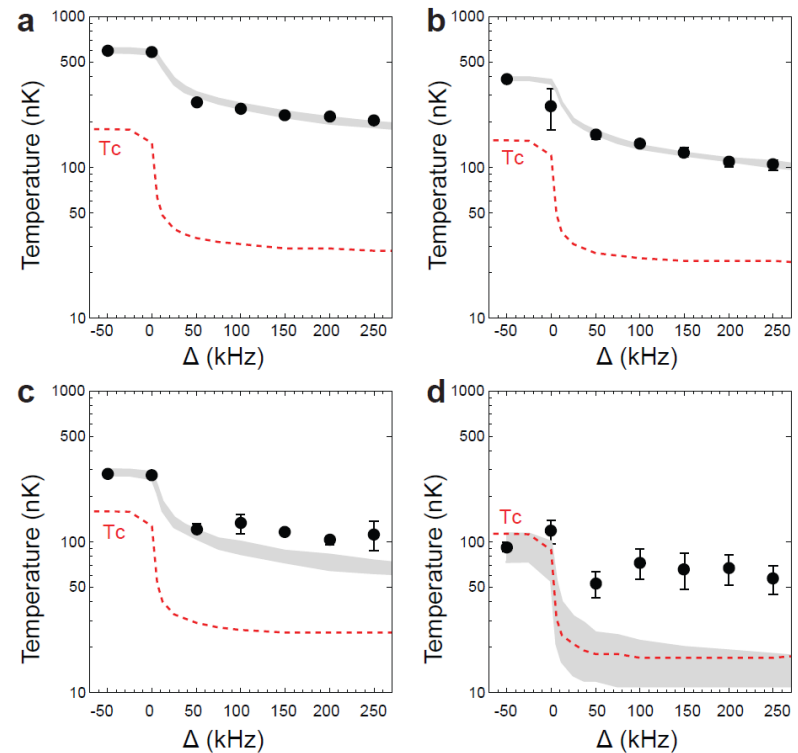
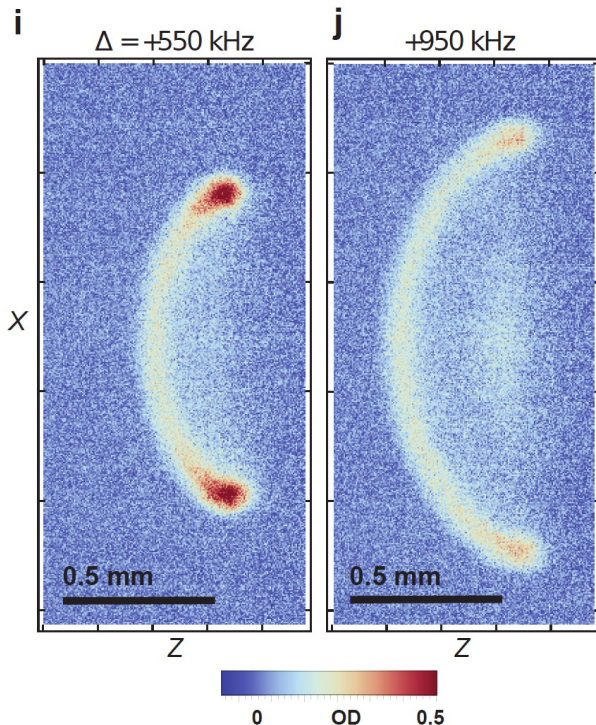
B. Rhyno et al, PRA (2021)

Related: Tononi and Salasnich, PRL (2019) ; Talk by Salasnich

CAL Bubble Experiment and Thermometry

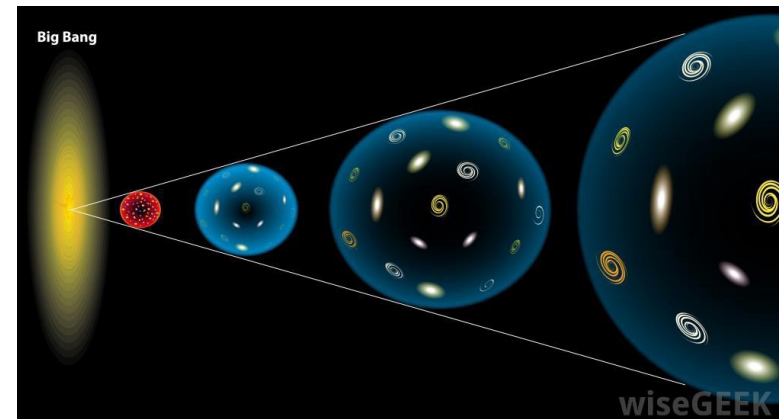
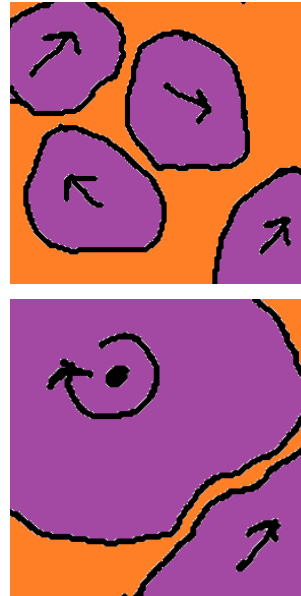
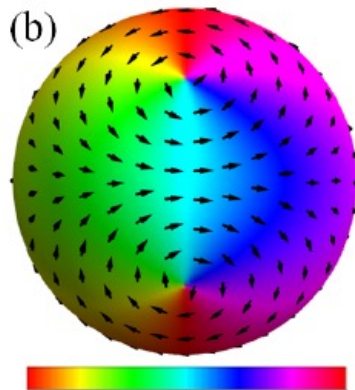
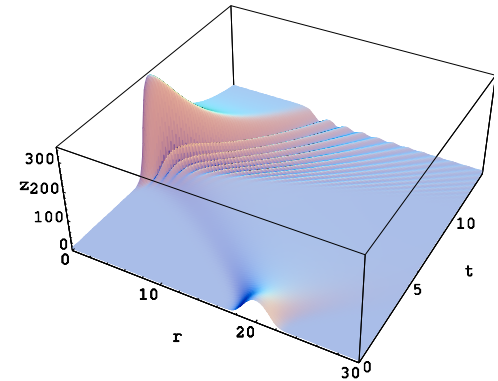
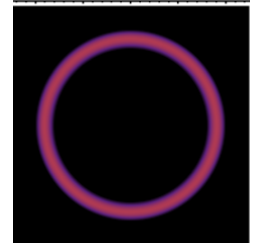


- Extreme inflation to mm-scale sizes ($T_i \approx 1\mu\text{K}$)



Future Directions

- Realistic traps and further thermometry
- Testing collective modes and expansion predictions
- BKT physics and vortex dynamics
- Coherent states and lowest Landau levels
- Non-equilibrium and Kibble-Zurek physics
- Expanding Universe



Many thanks to



***Courtney Lannert and group
Smith College***



***Nathan Lundblad and group
Bates College***



***David Aveline and colleagues
JPL***



***Kuei Sun
UT Dallas***



***Karmela Padavic
Bard HSEC***



***Brendan Rhyno
UIUC***

***Roman Barankov, Ryan Carollo, Brian DeMarco, Ethan Elliot, Joseph Murphee,
Robert Thompson, Tzu-Chieh Wei, Jason Williams, Frances Yang,***

And to you all!!



In Conclusion:

- Condensate shells in different contexts
- Superfluid-Mott systems—realization, collective modes
- Shell expansion
- Connecting with CAL Experiments
- Bubble trap collective modes
- Vortices
- Thermometry

