BEC-supersolid-quantum droplets transition in dipolar condensates in a ring potential

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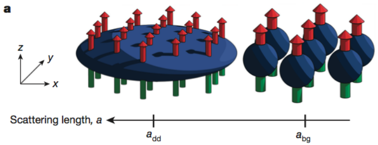
Introduction

- Experiment of Pfau group in 2016 observed spontaneous transition from an unstructured superfluid to an ordered arrangement of droplets in ^{164}Dy BEC

  Kadau et al., Nature **530**, 194 (2016)



- Quantum ferrofluid exhibiting Rosensweig instability

  Rosensweig, Ferrohydrodynamics (Cambridge Univ. Press, 1985)

- Local stability analysis shows that three-body interactions are not sufficient to describe droplet formation

  Ferrier-Barbut et al., PRL **116**, 215301 (2016)

- Quantum fluctuations remain as a main effect

Quantum droplets and supersolidity

- Dipolar droplets are predicted and shown to be self-bound

  Baillie et al., PRA **94**, 021602 (2016); Schmitt et al., Nature **539**, 259 (2016)

- Droplet structures or macrodroplets

  Chomaz et al., Phys. Rev. X **6**, 041039 (2016)

- Roton mode observation

  Chomaz et al., Nat. Phys. **14**, 442 (2018); Schmidt et al., Phys. Rev. Lett. **126**, 193002 (2021)

- Supersolidity in droplet arrays

  Böttcher et al., Phys. Rev. X **9**, 011051 (2019); Chomaz et al., Phys. Rev. X **9**, 021012 (2019)

- Very useful review: Böttcher et al., Rep. Prog. Phys. **84**, 012403 (2021)
- Geometry: 1D $\longleftrightarrow$ 2D
- Here: ring-trap

Quantum fluctuations: Bogoliubov theory

- We first consider spatially homogeneous case
- Shift of the chemical potential:

$$\Delta\mu = \frac{32}{3} g n \sqrt{\frac{a^3 n}{\pi}}\, \mathcal{Q}_5(\epsilon_{\mathrm{dd}})\,, \quad \mathcal{Q}_l(x) = \int_0^1 du\, \{1 - x + 3xu^2\}^{l/2}$$

Lima and Pelster, PRA **84**, 041604(R) (2011); PRA **86**, 063609 (2012)
Lee, Huang, and Yang, Phys. Rev. **106**, 1135 (1957)

- This does not take into account condensate depletion:

$$n = n_0 + \Delta n\,, \quad \Delta n = \frac{8}{3} n \sqrt{\frac{a^3 n}{\pi}}\, \mathcal{Q}_3(\epsilon_{\mathrm{dd}})$$

Quantum fluctuations: Bogoliubov-Popov theory

- Condensate depletion:

$$\Delta n = \frac{8}{3} n_0 \sqrt{\frac{a^3 n_0}{\pi}}\, \mathcal{Q}_3(\epsilon_{\rm dd})$$

- Shift of the chemical potential:

$$\Delta\mu = \frac{8}{3} g n_0 \sqrt{\frac{a^3 n_0}{\pi}} \left\{ 4\mathcal{Q}_5(\epsilon_{\rm dd}) + \mathcal{Q}_3(\epsilon_{\rm dd}) \frac{V^{(\rm int)}(\mathbf{q}=0)}{g} \right\}$$

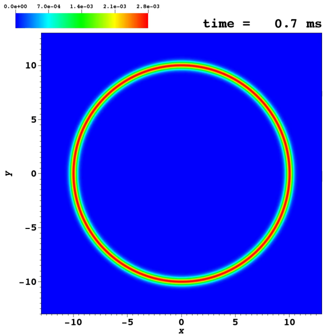
- Effective GP equation with LDA:

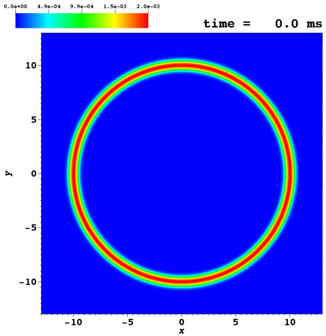
$$i\hbar \frac{\partial \psi(\mathbf{r},t)}{\partial t} = \left[-\frac{\hbar^2}{2m} \triangle + V_{\rm trap}(\mathbf{r}) + g n(\mathbf{r},t) + \int V_{\rm dd}(\mathbf{r}-\mathbf{r}') n(\mathbf{r}',t) d\mathbf{r}' + V_{\rm eff}(\mathbf{r},t) \right] \psi(\mathbf{r},t)$$

where $n_0(\mathbf{r},t) = |\psi(\mathbf{r},t)|^2$ and

$$n(\mathbf{r},t) = n_0(\mathbf{r},t) + \frac{8}{3}\sqrt{\frac{a^3}{\pi}}\, \mathcal{Q}_3(\epsilon_{\rm dd}) n_0(\mathbf{r},t)^{3/2}, \quad V_{\rm eff}(\mathbf{r},t) = \frac{32}{3} g \sqrt{\frac{a^3}{\pi}}\, \mathcal{Q}_5(\epsilon_{\rm dd}) n_0(\mathbf{r},t)^{3/2}$$

Droplet formation: low density case



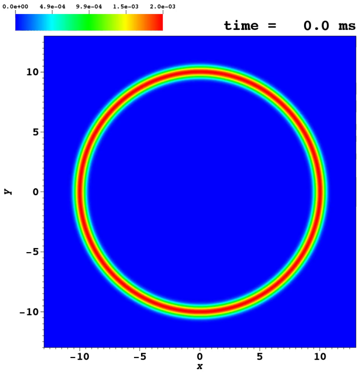


$N = 15\,000$ $N = 30,000$

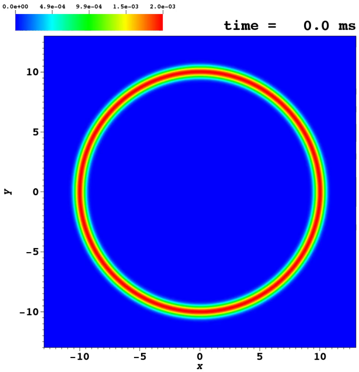
$$a = 132a_0 \to 62a_0,\ {}^{164}\text{Dy},\ a_{\text{dd}} = 132a_0,\ \omega = 2\pi \times 600\ \text{Hz},\ R = 10\ \mu\text{m}$$

Kumar et al., Comput. Phys. Commun. **195**, 117 (2015)
Lončar et al., Comput. Phys. Commun. **200**, 406 (2016); *ibid.* **209**, 190 (2016)

Effects of contact interaction quench size (1)

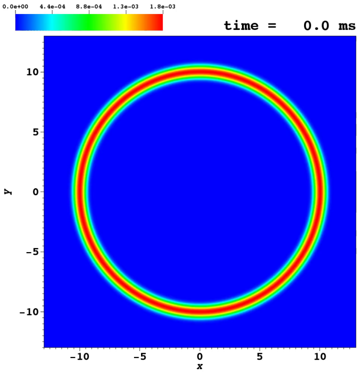


$$N = 30,000$$
$$a = 132a_0 \to 62a_0$$



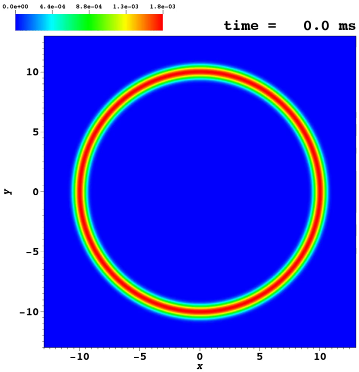
$$N = 30,000$$
$$a = 132a_0 \to 72a_0$$

Effects of contact interaction quench size (2)



$N = 50,000$

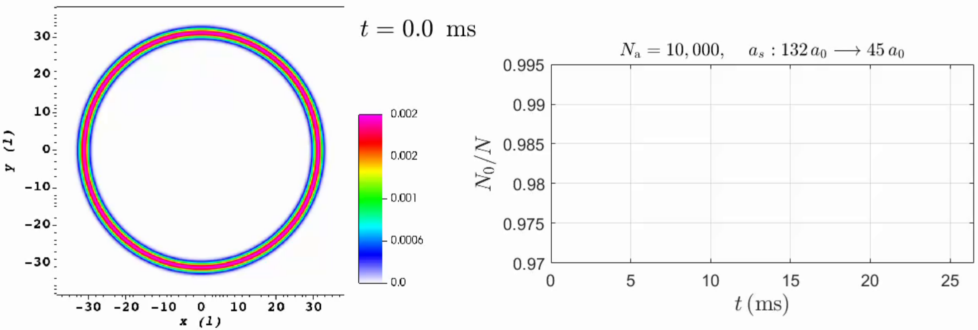
$a = 132a_0 \to 62a_0$



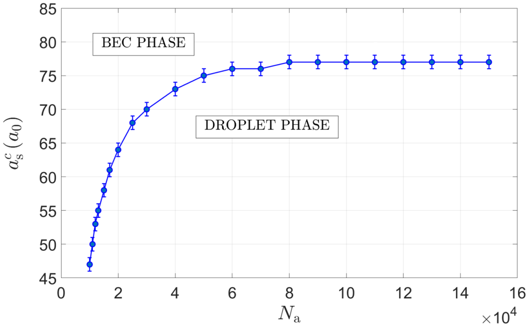
$N = 50,000$

$a = 132a_0 \to 72a_0$

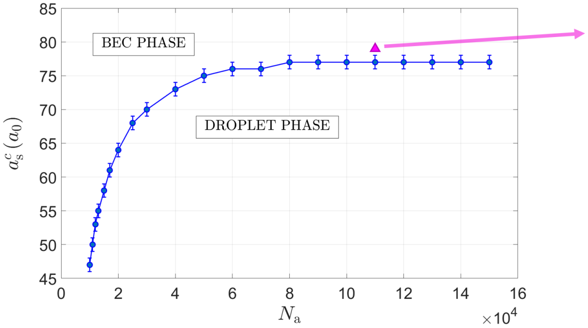
Condensate depletion

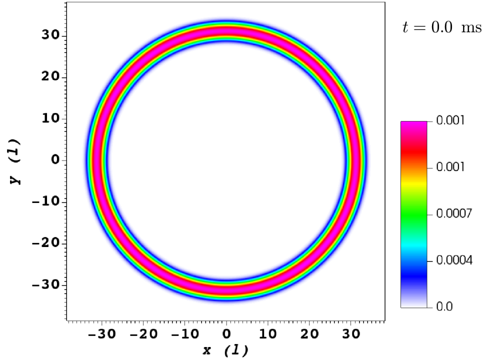


Phase diagram



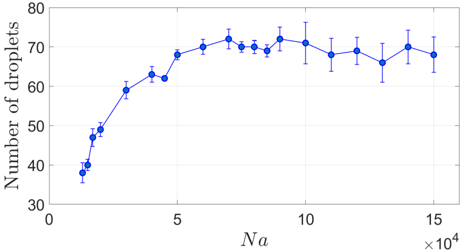
Phase diagram



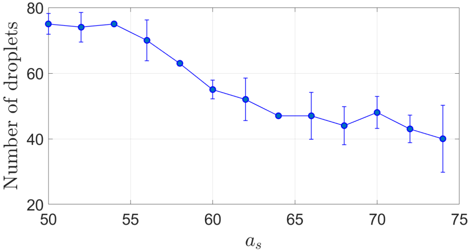


Number of droplets

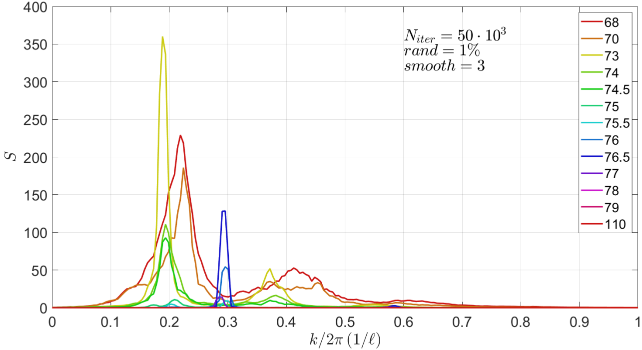
$a_s = 132\, a_0 \longrightarrow 55\, a_0$



$N_{\mathrm{a}} = 60 \times 10^3$

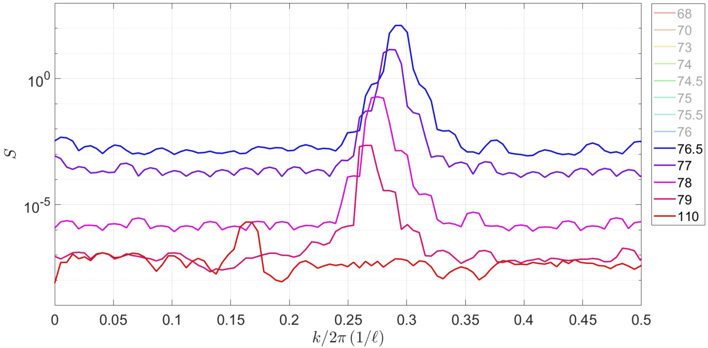


Structure factor from the ensemble average

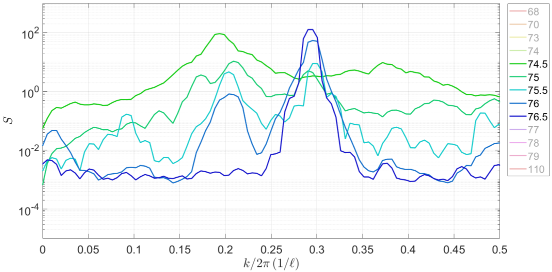


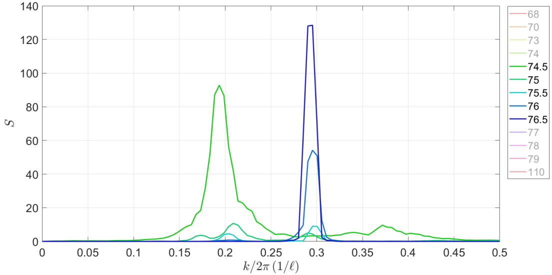
$$S(k_\varphi) = \left\langle \left| \int_0^{2\pi} \delta n_{1\mathrm{D}}(\varphi)\, e^{-ik_\varphi R\varphi} R\, d\varphi \right|^2 \right\rangle, \quad \delta n_{1\mathrm{D}}(\varphi) = n_{1\mathrm{D}}(\varphi) - \langle n_{1\mathrm{D}}(\varphi) \rangle$$

BEC phase

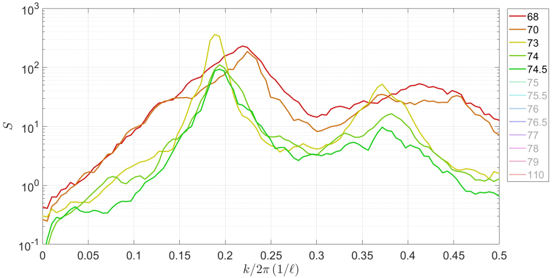

S
10^{0}
10^{-5}
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
$k/2\pi\,(1/\ell)$
68
70
73
74
74.5
75
75.5
76
76.5
77
78
79
110

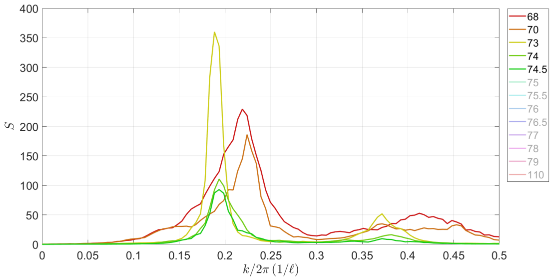
Supersolid phase



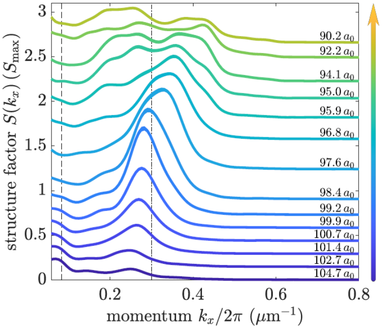


Isolated droplets

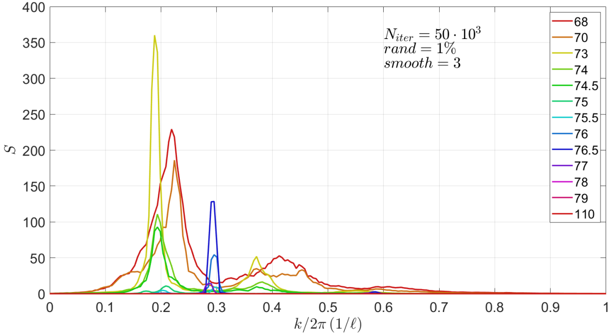




Structure factor: 1D vs. ring trap



Hertkorn et al., Phys. Rev. X **11**, 011037 (2021)



Conclusions

- Study of droplet formation in ^{164}Dy BEC in a ring trap
- Quantum fluctuations are the main effect responsible
  - Bogoliubov-Popov theory accounts for condensate depletion
  - Relevant when condensate density is high enough
- Conditions for droplet formation:
  - Sufficiently large contact interaction quench
  - Sufficiently high condensate density
- Phase diagram: BEC $\longrightarrow$ supersolid $\longrightarrow$ isolated droplets
- Characterization of phases via structure factor
  - Softening of the modes
- Number of droplets vs. number of atoms and contact interaction quench size